IS PARTICLE MASS A FUNCTION OF DEGREES OF FREEDOM?

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ABSTRACT

A formula is given for the ratio of the rest mass of the proton to the rest mass of the electron, leading to a formula for the ratio of the rest mass of the neutron to the rest mass of the proton, and a formula for the ratio of the mean rest mass of pions to the rest mass of the electron. The ratio of the lunar day to the Earth day is seen to have a similar formula. Numerous natural constants are shown to have commonality, and formulae are given for the derivations of their numerical coefficients, including the Rydberg Unit, the Fine Structure Constant, the Planck Constant, and the Boltzmann Constant. Ideas suggested by Eddington are expounded and shown to have further applications. A formula is given for the Earth's mean distance from the Sun, and also for ratios of other planetary mean distances from the Sun.

KEY WORDS: Proton mass, pion mass, neutron mass, commonality between natural constants, degrees of freedom of electron, lunar motion, planetary distances

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Many scientists have wondered why the rest mass of the proton was 1836 times the rest mass of the electron. Did that number have any significance? What could it possibly mean? Or was it 'just one of those things'?

As Paul Dirac said in a paper he delivered to the Cambridge Philosophical Society in May, 1930:

'The large difference between the masses of the proton and electron forms an unsolved difficulty in the existing theory. This large difference seems to be connected with the interaction between electrons, but our present ideas about interaction are inadequate to account for it. For this reason we cannot do better at present than to neglect the interaction altogether, which entails working with a theory in which the electron and proton have the same mass. This means, of course, a serious deficiency in our work and prevents one from attaching much physical importance to the result.'[1]

Anyone thinking deeply upon the matter of the mass ratio between the two particles would have realized that any answer would somehow have to be connected with the Rydberg Constant. But how? Or was it just a vain hope that there might be some rational connection or explanation for those mysterious

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dimensionless numbers which are so fundamental to the whole of physics, namely the ratio of the rest mass of the proton to that of the electron, and the numerical discoveries of the brilliant Johannes Rydberg?

I have found a possible answer to this question, which, as one would expect, is numerical. The number which sprang forth in answer to this problem was the number 136, which, as Eddington stressed in three of his books, commencing with *New Pathways in Science*,[2] is the number of 'degrees of freedom' of the electron.[3] He gave a fuller account of this in his book *Relativity Theory of Protons and Electrons*.[4] In that book he uses language more familiar to contemporary scientists by speaking of a single system S_0 consisting of the combination of two particles *S*, *S'*, and with Θ being the phase variable of *S*, he says: 'The phase space of S_0 contains 136 coordinates $\Theta_{\mu\nu}$. … We have to take a system represented by a double wave vector in any combination of its 136 phases, all being relativistically equivalent …' Or, as he puts it more simply: 'Initially the probability of S_0 is distributed uniformly over its 136-dimensional phase space.'[5] This kind of language may possibly be found to be more comfortable to scientists of today than the more picturesque language of '136 degrees of freedom'. Eddington's final, extremely elaborate, and most mature reflections and comments on this subject appear in his lengthy posthumous work, *Fundamental Theory.*[6]

Before discussing further the other occurrences and significance of this number, 136, I hasten to show the formula by which it can be shown to be the possible basis for the number giving the rest mass of the proton. Johann Jacob

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Balmer's fundamental formula which preceded the discoveries of Rydberg was a rather mysterious and unexpected leap forward for physics in 1885. Rydberg then made his breakthrough on the back of that in 1898. I shall survey the fundamental importance of 136 in the Rydberg calculations in a moment. The first thing to do before giving the formula for the rest mass of the proton is, however, to come up with a symbol, just as Balmer had to come up with R and Rydberg had to come up with *R*∞. I have tentatively adopted as a symbol for 136, in tribute to Rydberg, the accented letter *ř*. The rest mass of the proton can thus be expressed as follows by a simple formula:

$$(\check{r}^2)^{-1} - \check{r}^{-1} = m_p \qquad (1)$$

Before trying to think what the implications of this might possibly be, it is necessary to show the formula's affinity with the findings of Johannes Rydberg, and to combine those with a universal correction factor whose importance has in recent years been emphatically stressed by Martin Rees, but which has apparently never before been applied to Rydberg's computations.

The numerical significance for which we seek is not found in the standard expression of the Rydberg Constant, but in its alternative form known as the Rydberg Unit (which appears as *R*y in equations), which is derived from it. Looked at from another point of view, one might claim that the Rydberg Unit is the fundamental form and that the Rydberg Constant is the derived form, though that is not the sequence of their discovery. It is not necessary to insist upon this, but I mention it as a possible viewpoint.

In 1916, Arnold Sommerfeld discovered the Fine Structure Constant, often written as 137, a coupling constant which characterises the strength of the electromagnetic interaction. According to the numerical computation of this coupling constant done in 2011, the true value is 137.03599074. (Hence the Fine Structure Constant = ř + 1.03599074.) Using this value, we find that if we divide it by the Rydberg Unit of Energy times ten (*R*y x 10), then the result is that the two coefficients are equal to one another to within .007. This is so close that a coincidence would seem to be unlikely. And indeed there is forthcoming a possible reason for this, which is that this precise number, 0.007, governs the process of the formation from hydrogen of all chemical elements in the Universe. In his book *Just Six Numbers*, Martin Rees states: '… what is so remarkable is that no carbon-based biosphere could exist if this number had been 0.006 or 0.008 rather than 0.007.'[7] So now we see this very same number occurring as an apparent correction factor, in the form of a supplementary natural constant, as a means of adapting the number which specifies the Rydberg Unit so that it can apply to electromagnetism in the form of the Fine Structure Constant.

Seeking for other occurrences of *ř* in connection with electrons, such as in distinguishing isotopes, we may turn our attention to Uranium 235 and Uranium 238. When expressed as a ratio, the numbers 235 and 238 differ from one another by 0.0128, which is within 0.00075 of 0.0136. I shall have more to say about 136 occurring as a decimal fraction in a moment.

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If we turn our attention to other aspects of chemical elements, let us consider the temperature at which Helium^4 liquifies and compare it with the temperature at which Helium^3 liquifies. The first liquefies at 4.2216K and the latter liquefies at 3.2K. The mean between the two is 3.7K, and that value is precisely 0.0136th of the boiling temperature of water.

These occurrences of 136 in decimal fractions may seem at first glance to be fortuitous. However, one may look at them in a different light when one is as familiar as advanced musicologists are with the unique and fundamental number 1.0136, which from their point of view is most definitely a fundamental and universal natural constant. One must remember that sound is part of physics, and although few physicists since von Helmholtz have made such deep and detailed studies of music in the context of physics, it should never be forgotten that despite the fact that musical theory is usually left to musicologists, it need not necessarily be so. Ever since ancient times, it has been known that there was a 'problem' with music, and that that 'problem' was expressed by a specific number. In antiquity, it was normally written as a fraction, but in decimal terms, the number is 1.0136. This number, carried out accurately to an astonishing nine decimal places in the form of a fraction, is preserved in the ancient Pythagorean treatise *Katatomē Kanonos* (*Division of the Canon*).[8] (This work was only edited and published in translation for the first time in 1991, and is known to very few classical scholars because of its extreme arithmetical technicality.)

This number, 1.0136 in its decimal form, has been known for well over two thousand years in its fraction form. Its name is the Comma of Pythagoras. It

was named, obviously, in honour of Pythagoras, and the implication is that he personally knew about it. In fact, there are good reasons for suspecting that the number was known long before the time of Pythagoras, but this is not the place for a discussion of such ancient matters. What does the Comma of Pythagoras mean? It represents the incommensurability of two separate arithmetical sequences used in musical harmony. People unfamiliar with music theory will be surprised to learn of this. If you count upwards (or downwards) on the keyboard by octaves, you will reach a point which is close to but not identical with a point if you count upwards (or downwards) by fifths. You discover to your surprise (if you have a means of measuring frequencies accurately, that is) that there is a tiny arithmetical discrepancy between the mathematics of the system of octaves and the mathematics of the system of fifths. This discrepancy makes it impossible to modulate between keys because of the appalling discords which result. The ear can instantly detect the problem. The system known as equal temperament (promoted in the work by J. S. Bach entitled *The Well-Tempered Clavier*) overcomes this difficulty by inventing artificial semi-tones and shaving something off each note, so that each note is slightly flat. It is only this mathematical compromise which has made it possible for music since Bach to modulate between keys and thus render possible such works as Beethoven symphonies, not to mention the even more complex multi-key compositions of Rachmaninoff, Bruckner, Mahler, and others. All the above mentioned compositions are slightly flat, which the ear of a musician with perfect pitch can detect, but most people are unaware of the sacrifice of 'pure tone' which has to be made to overcome the number 1.0136.

The decimal component of 1.0136 can be detached and the number 136, at whatever power of ten one chooses, treated separately as a pure number. Since it has until now lacked a name, I have named the decimal portion 'the Particle of Pythagoras'. The numerical value of the Rydberg Unit of Energy is 13.60569253, which is simply a matter of slipping the decimal point along one power of ten, with an increment as shown in the decimal fraction.

The neutron to proton mass ratio is 1.001378419. Later in the discussion we will see the significance of 137 as compared with 136, but it should be noticed that we here have the number 137 (a variant of 136, as subsequently explained) shown as a decimal increment describing the relationship between protons and neutrons. The ratio of rest mass of a neutron to that of a proton could thus be described by a dimensionless number as being $(\check{r} + 1)^{-3}$.

The mean rest mass of pions in MeV/c^2 is 137. That is because π^0 = 134.97; π^+ and π^- = 139.57.

The ratio of the rest mass of the pion$^+$ and the pion$^-$ to the rest mass of the electron is 273 = $2\check{r} + 1$.

$(273)^{-4}$ is the decimal increment ('mantissa') of the square of the Comma of Pythagoras (i.e., $1.0136^2 = 1.0273$), and 273^o Kelvin is zero degrees Centigrade, as I discuss below.

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If we treat *ř* as a numerical coefficient which can occur at any power of ten, then we discover that the density of liquid mercury is 13.6 times the density of water, and the number *ř* is thus found again. This may be coincidence, or it may be significant. It is the second time that water has arisen in the search for manifestations of *ř.* That too may be a coincidence, or it may be significant. At this stage, it is not yet clear what is significant and what is not. When the role of *ř* is better understood, some of its occurrences may be found within the context of a larger understanding to be trivial manifestations rather than fundamental ones.

Clearly, one feels uneasy about such an apparently important natural constant as *ř* unless one can relate it to some equally fundamental natural constant which is familiar to everyone. I therefore spent a great deal of time examining this question and did not rest until I had found a connection with *pi*. Once I had connected *ř* with pi, I felt that I could be less anxious about the novelty of putting forward what might appear, despite its ancient pedigree, to be a new natural constant, at least to our eyes today. The connection with pi is as follows.

If we take the decimal increment of pi and the decimal increment of the inverse of *ř*, then we discover that they have a relationship. This may seem a strange way of thinking, but it was inspired by the concept of logarithms, where the decimal increments of numbers are called mantissas and are dealt with separately. Using the language of logarithms, I thus refer to 0.1416 as the 'mantissa of pi*'.* This is just a way of speaking, not a definition, but I have

sometimes found it useful to detach the decimal increments of natural constants in this way.

The rest mass ratio of the neutron to the proton is 1.00137. In other words, $m_n/m_p = 1.00137$. The number 137, which is the fine structure constant, is something which is discussed further below, where its relationship with the number 136 is made clear. For the purposes of this discussion I represent 137 as $\check{r} + 1$. Here we see the number 137 as the numerical coefficient in a 'mantissa'. A formula for the rest mass ratio of the neutron to the proton is:

$$m_n / m_p = 1 + (\check{r} + 1)^{-3} \qquad (2)$$

Sir Arthur Eddington believed that he had discovered a number which was the 'universal uncertainty constant', the value of which he stated was 9.604×10^{-14} (operating at the dimensions of nuclear particles, as that particular power of ten shows).[9] I shall have more to say concerning that number later, when discussing Eddington's theories more fully. If we take this number of his and treat it as a numerical coefficient operating at any power of ten, and if we express it as 0.09604, we find that if we multiply 0.1416 (the 'mantissa of pi') by it, it gives us precisely 0.0136, or the Particle of Pythagoras, i.e., $\check{r}^{-4}$. There thus seems to be a relationship between pi and $\check{r}$ by means of Eddington's constant, which he represented in his equations by the Greek letter *sigma*. The formula of this is therefore:

$$\check{r} = (\pi\text{-}3) \times (\sigma_{coef})^{-2} \qquad (3)$$

Perhaps this too is fortuitous, but seeing three natural constants together in one formula may be significant. After all, how often does that happen? Just in case one might be tempted to criticise these findings, one should remember that it was precisely by thinking in terms of dissociating the decimals and calling them mantissas that John Napier created his system of logarithms. If he can do something productive by taking that approach, why cannot we? It may seem odd, but then, John Napier was odd, and so was his discovery. Perhaps therefore there is 'something in it'. All three of the natural constants in the above formula have been treated as numerical coefficients that can operate at any power of ten, and the detachability of the 'mantissas' of two of them have thus been found to have an arithmetical relationship with each other, precise to four decimals, when intermediated by a universal 'uncertainty' coefficient discovered by Eddington.

Let us just take a moment to reflect on how this discussion started. It began with 'the number of degrees of freedom of an electron', and that led us to the proton. Is there any specific theory which has been suggested by modern physicists to suggest that the proton really is somehow derived from the electron? There was a suggestion made by Yoichiro Nambu (1921-2015) and Giovanni Jona-Lasinio relating to this. In their famous early two-part paper 'Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity' published in the *Physical Review* in 1961, they stated:

'… we would like to regard our theory as dealing with nucleons and mesons. The implication would be that the nucleon mass is a manifestation of some unknown primary interaction between originally massless fermions …'[10]

They were clearly not referring to composite fermions, and must therefore have been thinking of electrons which they thought of as having been originally massless, a not unknown concept which is from time to time mentioned as a theoretical possibility, especially in the contexts of discussions of velocity-related relativistic mass variations of particles (hence I have taken care to commence this article with specific reference to 'rest masses' of the electron and the proton).

I have not undertaken a review of the later work and theories of Nambu and Jona-Lasinio, and leave that to those who are more familiar with the subject. But I wanted to cite their early suggestion, as it is so tantalizing.

With regard to this curious concept that protons could have arisen from a massless source, I am reminded of a strange statement made by the normally very conservative Werner Heisenberg in one of his short treatises:

'… the best description … is … to speak about the creation of particles out of energy, according to the laws of relativity. We can say that all particles are made of the same fundamental substance, which may be called energy or matter; and to formulate: the fundamental substance "energy" becomes "matter" by assuming the form of an elementary particle.'[11]

Clearly, some great minds have been struggling with these issues in a heroic manner but have reached no firm conclusion.

We thus face a serious issue of: *what is mass, or is it anything at all*? Newton and Einstein agreed on one thing, which was that inertial mass and gravitational mass must have the same value, or perhaps 'measure' is the better word. But since Einstein, numerous speculations about mass have appeared, including such strange suggestions as the existence of 'imaginary mass' and even 'negative mass'. Where is all this leading us? If one reads the second revised edition of Max Jammer's famous work, *Concepts of Mass*, one can easily become befuddled by the multitude of varying views. At the end of his book, Jammer informs us:

'None of the theories of mass discussed thus far, whether global or local, has ever gained general acceptance, for a number of reasons. First of all, none of them predicts the masses of the elementary particles.'[12]

So nobody yet knows what mass is, and Jammer admits it. Note that he talks about the existing theories being unable to predict the masses of the elementary particles. The ideas set forth in this article explain the mass ratio between the two major elementary particles. So perhaps that is an advance.

Assuming that the connection between particle mass and 'degrees of freedom' has the importance which seems to be suggested by what has been said

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above, it then becomes urgent to begin thinking of what the theoretical significance of that might be, for it could be considerable. When thinking along these lines, I have found it helpful to consider these remarks made by Mary Hesse in her book *Forces and Fields*:

'The phenomena of pair-production suggest that there are continual real and virtual creations and annihilations of photons in the field, the distinction between real and virtual being essentially a distinction between photons that are *free* and those that are *bound*, for since real radiation has definite wavelength and hence momentum, there is a finite probability of finding it anywhere in space, whereas virtual photons are bound to a limited region of space surrounding their point of creation, since if they had time to get far, observable energy conservation would be violated. Consequently the momenta of bound photons is to the corresponding degree uncertain. It is found that if the lowest energy level of a field with static source-particles (positive or negative charges) is considered, this corresponds to a field containing only bound photons (the *vacuum field*), and that the actual energy of the virtual photon-exchanges is the Coulomb energy of the source-particles. This exchange process cannot be understood to be virtual in the sense of being merely *potential*, for clearly if A is in possession of a ball without *actually* throwing it to B, no force acts between them. "Virtual" has always to be understood as non-energy-conserving, and therefore short-time.

'In a similar way, nuclear forces can be represented as virtual exchange of mesons, and here the force between like sources may be attractive, so that the

theory overcomes the difficulty which Maxwell found for the case of gravitation, in accounting for gravitational *attraction* rather than *repulsion* by means of a field similar to the electromagnetic field. The reserves of energy in the vacuum field which Maxwell found necessary but implausible, are available in the meson field. These potentialities of the general field theory naturally lead to the hope that the gravitational field can be shown to be a particular case of interaction by means of virtual particles, either particles peculiar to gravitation, or particles already recognised in electromagnetic and nuclear theory. Such a theory in flat space-time might be alternative to the replacement of gravitational force by a Riemannian metric as in general relativity.' (She here refers to articles by Robert H. Dicke of 1957 and Harold Albert Wilson of 1921.)[13]

It is immediately clear that the discussion of 'photons that are free' *vis a vis* 'photons that are bound' is effectively a discussion relating to degrees of freedom. For what else does 'free' mean other than that one or more degrees of freedom is enjoyed?

It is by no means impossible that by reconceptualising numerous phenomena of physics from the 'degrees of freedom' point of view, one could understand gravitation, electromagnetism, nuclear forces, and indeed the whole of cosmology in a new way. And one could do so with the help of a new comprehension of universal dimensionless numbers, 136 and 1836, which is always such a comfort. After all, Planck's constant is a dimensionless number, and look what it led to. The precision of thought to be gained from new dimensionless numbers entering the picture is such that experiments should

occur to working scientists, in order to substantiate or to falsify theoretical insights emanating from this development. For instance, it should be possible to investigate more closely in new ways some of the phenomena relating to mesons as intermediaries. I venture to suggest that mesons might be phenomena mediating between 'free' and 'bound', that they represent brief intermediation phenomena or processes relating to the increase and decrease of numbers of degrees of freedom of particles. It must be possible to test this somehow. We must remember that we have already seen that the mean mass of pion mesons is 137, which is a suspicious recurrence of the number 137 in precisely the place where we would most expect it, namely with mesons. Pions can 'decay into electrons'. Perhaps we should try to reconceptualise what is really going on when that happens. Pions have zero spin and electrons half one-half spin. When electronic decay of π^+ and π^- pions is suppressed, it is a spin effect called *helicity* suppression. We will have more to say about helicity later on. The connection between helicity and the phenomena of spin is an interesting area for exploration. But to return to the supposed decay of pions into electrons, which was apparently discovered in 1958 at CERN, what happens to all the missing mass, if electrons have a rest mass of 1 compared to the proton? It seems we are using different scales. But according to one scale, are we to consider a loss of 136/137 of a pion's mass resulting in the transfer to the newly created electron of 136 'degrees of freedom'? What is the connection between 'mass' and 'freedom'? Surely mass is a loss of freedom? Can the same number describe the loss of mass by a mediating particle and the gain of freedom by a resultant particle? The only other decay product appears to be a massless neutrino known as an electron antineutrino. So where does the mass go, and where does the

freedom come from, and why does the same dimensionless number apply to both?

Before Nambu and Jona-Lasinio published their idea of a theory 'dealing with nucleons and mesons', Mitra and Saxena had already published an intriguing article about meson-meson interactions, pointing out that they are impossible to study directly by the scattering of one meson beam by another. They comment upon the 'meson cloud' surrounding every nucleon. Certainly the fact that nucleons are surrounded by meson clouds coincides very well with what has been suggested above. Here is a comment which they made about this matter:

'The concept of meson-meson interaction (which can be pictured as taking place through virtual nucleon pairs) is almost as old as that of the more fundamental meson-nucleon interaction. However, unlike the latter which can be realized directly by means of a meson-nucleon scattering experiment, the former finds only indirect verification by the effect it produces on a system consisting of two or more mesons, since it is not possible to conduct any direct experiment for scattering a meson beam by another. One of the most important processes in which π-π interaction plays an indirect role is one involving scattering of mesons by nucleons, if it is remembered that a nucleon is always "dressed" with its own meson field which the incident meson beam has to encounter. Strong resonant interactions of the meson beam with the meson-cloud of the nucleon were in fact suggested phenomenologically by Dyson and Takeda … Though the importance of the π-π interaction influencing various physical phenomena has been generally

recognized, a field-theoretical treatment of this interaction has hardly received any attention so far.'[14]

It was Hideki Yukawa in 1934 who originated the theory of meson fields and postulated that the strong nuclear force holding nucleons together was borne by mesons, and detected that there was a relationship between the range of the force and the mass of the force-bearing particle, or meson. It is the π-mesons, generally just called pions, which are the most important of the mesons. I have already pointed out above that the mean rest mass of pions is 137, which is equal to ř + 1, and the ratio of the rest mass of both the pion$^+$ and the pion$^-$ to the electron is 273, which is equal to 2ř + 1. Clearly we have, with pions, two manifestations of the 136 degrees of freedom. The 'force' postulated by Yukawa must therefore be a transference of these degrees of freedom between nucleons and electrons. This goes some way towards defining the 'relationship' which Yukawa intuited between the force borne and the mass of the bearer of the force.

Fermi says: 'The force field of a nucleon extends as far as the pions which it continuously emits and reabsorbs are able to reach.'[15]

More generally, he gives this summary of Yukawa's original conception:

'The Yukawa theory of nuclear forces is based on the assumption that the forces acting between two nucleons are transmitted by the pion field that surrounds them. The phenomenon is usually described by stating that when two nucleons approach each other one of them may emit a pion which is absorbed by

the other. … Phenomenologically the transition from initial to final state … could be described as due to a force acting between the proton and the neutron without explicit reference to the role played by the emission and absorption of the pion. This force, however, is represented not by an ordinary potential but by a so-called exchange potential. In simple terms this can be understood by observing that in the emission of the positive pion the original proton changes to a neutron while in the subsequent absorption of the positive pion the original neutron changes to a proton so that the role of the two nucleons is interchanged.'[16]

There's that word *interchange* which we shall shortly encounter stressed by Eddington as being fundamental to his theories.

A complementary way of regarding the strong nuclear force, which is apparently 100 times stronger than Coulomb forces, is that it could be an example of the concept so central to electronics, namely *amplification*. Amplification in electronics turns a weak signal into a strong signal, and it does not necessarily require an amplifier, such as we in our macroscopic world employ, because it apparently occurs spontaneously at the ultra-weak level in Nature in electron streams under the influence of magnetism. What therefore is there to prevent the strong nuclear force being merely an amplification of Coulomb forces within the special contexts of nuclei? And could the amplification be connected with the releasing of some 'bound' energies in the forms of the releasing of additional degrees of freedom? Is this what mesons are really up to? Is the 'force' carried by a pion really an amplified Coulomb force,

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which has been amplified by the mediation of what we have been calling the "degrees of freedom'? Are the pions really therefore acting a bit like travelling-wave tubes in electronics? If so, then we would certainly expect to encounter helicity, because travelling-wave tubes can only operate on the basis of the numbers of turns in the helix inside each one. In a travelling-wave tube, the number of turns of the helix has a linear relationship with the amount of amplification taking place. Does each pion therefore 'have' 136 helical turns composing its inner structure or path? The 137^{th} would be additional and correspond to 'target achieved', according to the reasoning of Eddington (who calls it *interchange*). Is the rest mass measure of a pion therefore nothing more nor less than a count of the number of turns of its inner helix? And in connecting with an electron, where the rest mass ratio is $2\check{r} + 1$, do we therefore have a double-helix in operation? The double-helix configuration was made famous when it was discovered inside the DNA molecule, but long before that was known, double-helixes were known to be the basic interior configurations of Birkeland Currents for the transport of charged currents over vast distances with minimum resistance. Such currents are dominated by pinch effects, and it was in 1934 that Willard Bennett discovered the 'Z-pinch', one form of which is now called the 'Bennett Pinch' after him.[17] Pinches turn plasma streams into plasmoids, among other things, and that is analogous to forming a 'corpuscle' or 'singular region' in an electromagnetic wave. (See the discussion below of Louis de Broglie's Theory of the Double Solution, for which he worked out all the mathematics to explain how to reconcile the wave-particle duality within the confines of quantum mechanics.) It seems obvious to me that there must be pinch effects going on at the particle level, that the pions are operating like microscopic

Birkeland Currents channelling charges along helices, and that 'pinches' occur when new particles are formed. If we think like this, we can look at the vast and richly complex field of particle physics in an entirely new way which suddenly gives sense to so much of what has appeared puzzling about it until now. It is not, however, my task to give particle physics 'a new, fresh look' which will make it appear so much more 'user-friendly' and so much less bizarre. That is for particle physicists themselves to do.

It is necessary to give a bit more information about 'pinches'. An interesting summary of their technological history was prepared in 2006 by the US Naval Research Laboratory, under the title of 'Z-Pinch Plasma Neutron Sources'.[18] Here is part of the historical summary:

'The idea of using deuterium Z-pinch plasma to generate fusion neutrons is not new. In fact, the quest for controlled thermonuclear fusion started in the early 1950s from experiments with deuterium Z-pinches. This research was launched back then in parallel, almost simultaneously, with the development of thermonuclear weapons, which employ artificial self-sustained fusion reactions on a much larger scale. Such weapons were first successfully tested in 1952-1953. The first nuclear fission weapon was tested shortly before that, in 1945, less than three years after E. Fermi brought on line the first artificial nuclear fission reactor. With nuclear fusion, it was the other way around. The bomb was built first, and the researchers were confident that a controlled fusion reactor was to follow in the not-too-distant future. The first observations of DD fusion neutron yields from linear Z-pinches reported by the U.S., Soviet and European

laboratories supported this optimistic view. Deuterium pinches were studied then as prototypes of magnetic confinement fusion reactors.

‘This early optimism, however, did not last long. It was soon understood that the observed fusion neutrons were not thermonuclear. Rather, the neutrons were produced in Z-pinch plasmas by relatively small quantities of “beam” deuterium ions accelerated in the direction of the current, in the strong electric fields accompanying the development of the “sausage” *m* = 0 instability of the pinch, to energies of 50-200 keV. Colliding with the deuterium plasma ions, whose temperature was much lower, the beam ions produced fusion neutrons, which were thereby not thermonuclear. … Since then, linear Z-pinches as magnetic confinement fusion devices have been abandoned for good in favour of toroidal systems, primarily tokamaks.’

That is the early history. The authors go on to say that Z-Pinch plasmas made a comeback in different ways and ‘are presently the most powerful and energy-efficient laboratory sources of X rays.’ Indeed, fusion generation has continued to be pursued in Z-pinches, especially the production of fusion neutrons. Those plasma neutron sources are now being used as triggers in combination with fissionable materials for multiplication of neutron yields. It is suggested that this can be part of the process for ‘a safe sub-critical fission-fusion reactor that can actually produce energy.’

The subject of Z-pinches and other pinches is a large one, much of it is not public, and the implications of pinches in cosmology is barely understood so far. But there is no doubting its importance.

I suppose I should come up with a name for this approach. Drawing upon the ancient Greek word for freedom, which is *eleutheria*, I propose as an initial suggestion that this approach might be called the eleutheric interpretation of quantum mechanics. It cannot be called a theory at this point because no experiments and tests have yet been formulated either to verify it or to falsify it.

Any particle, whether real, virtual, pseudo-, simple, complex, or of any other strange nature, which could be seen to be acting as an intermediary either to reduce or to increase degrees of freedom could therefore be named an *eleutheron*. I am not proposing that mesons should be renamed eleutherons, but that they might one day come to be recognised as forming part of a family which could be called the family of eleutherons (just as there are lepton and baryon families, and fermion and boson families). In other words, eleutherons would be a family of facilitators, whether or not they were really particles at all; they could be merely *processes*. We may have in the z-pinch effect in plasma a good example of an eleutheric process. Z-pinches are also known as Bennett pinches when they occur in charged Birkeland Currents, as a process by which encircling magnetic fields, generated by the currents themselves and orthogonal to the current flow of course, squeeze the currents to such an extent that significant effects occur, leading to exceedingly high temperatures capable in the space environment of nuclear fusion phenomena. This is not the place for a

discussion of this subject, but I wished to mention it in the context of potential eleutheric processes which are already familiar to us.

Central to such an eleutheric approach is the concept of mass, or should I say the *concepts* of mass. I have known numerous 'renegade' scientists personally, commencing with Paul Dirac, who introduced such initially disruptive but eventually productive concepts as half-spin and anti-matter into physics in an irrevocable manner. He told me in the 1960s that he was unhappy with the way physics was going because his colleagues would not listen to him when he kept urging them to use quaternions. It should be noted that quaternions make it easier to represent and understand rotation and orientation (both of which are central to concepts of degrees of freedom). Since the 1960s. their importance has been better realised and they have been adopted widely in certain fields, such as orbital mechanics of satellites and flight dynamics. Also, since the days when Dirac made those comments to me, non-commutative phenomena (which can be represented by quaternions) are universally accepted and as familiar to everyone as bread and water. All of this relates to questions of degrees of freedom. There is clearly a vast realm to explore when one considers only these aspects, and there are so many more besides.

To return now to the computations of Rydberg, we see in the equation defining his constant the symbol ε_0, which stands for 'the permittivity of free space' (also known as the vacuum permittivity or the electric constant) giving the value of the absolute dielectric permittivity of the classical vacuum. This is interpreted as 'the capability of the vacuum to permit electric field lines'. That

standard statement is obviously an 'as if' statement dealing with fictitious entities, since electric field lines do not exist. They are mathematical abstractions invented by us to act in a manner similar to contour lines on maps. What therefore does it mean to say that this constant relates to the freedom for the fictitious electric field lines to be permitted by a classical vacuum? We are dealing with *freedom*, as in 'degrees of freedom'. We are here face to face with a classic eleutheric concept. Will or will not the classical vacuum allow electric fields to manifest themselves (which we conveniently portray by means of lines which we draw)? The answer is that part of the formula for the Rydberg Constant directly concerns the freedom of electrons to become manifest, whether alone or in a current generated by a 'field' which we conventionally call 'an electric field'.

I put the words 'electric field' in quotes because many scientists have been at pains to stress that they do not really exist, any more than electric field lines exist. As Heil and Bennett put it in their textbook when commenting on field lines:

'It must not be inferred from what has been said that lines of force or induction exist in any real sense. They are not substitutes for electric fields or electric forces in any way. They are *mathematical conveniences for representing electric fields.*'[19]

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One finds a sharp rebuke to those who believe in the reality of fields themselves in the writings on the philosophy of science by Percy Bridgman, who won the Nobel Prize for Physics in 1946. Here are his comments:

'… I believe that a critical examination will show that the ascription of physical reality to the electric field is entirely without justification. I cannot find a single physical phenomenon or a single physical operation by which evidence of the existence of the field may be obtained independent of the operations which entered the definition. The only physical evidence we ever have of the existence of a field is obtained by going there with an electric charge and observing the action on the charge … It is then either meaningless to say that a field has physical reality, or we are guilty of adopting a definition of reality which is the crassest tautology. … Having now learned how to measure electrical quantities, we discover experimentally the inverse square law of force, and later arrive at the concept of the electric field. As we have seen, the field is an invention; here we shall use this concept only for the purpose for which the invention was made, and shall not involve ourselves in any of the implications of ascribing reality to the field. … Now we have already emphasized that the electromagnetic field itself is an invention, and is never subject to direct observation. What we observe are material bodies, with or without charges (including eventually in this category electrons), their positions, motions, and the forces to which they are subject. … The electromagnetic field as such is *not* the final object of our calculations … we can compute the electromagnetic field, overlooking the fact that the field has no immediate meaning in terms of experience.'[20]

If fields are a convenient fiction, useful in computations, then what is the reality of what is going on, as opposed to the fiction of what is going on? We can talk about fields all we like, and have elegant and useful solutions to equations by pretending that they exist. But that is a bit like kissing somebody in your sleep when you are alone in bed. For by postulating fields, we are grasping phantoms.

We need a new and better way of approaching the reality of what is going on which at the moment most or perhaps all scientists explain by speaking of fields. Back in the innocent days before children knew too much to believe such things anymore, the magical appearance of Christmas presents under the Christmas tree was conveniently explained by the fantasy of Santa Claus having come down the chimney in the night. (Those were the days when people had chimneys.) Also in the innocent days, when children asked parents where babies came from, mothers did not speak of their tummies, but instead would say things such as: 'The stork brings them,' or 'We find them in the cabbage patch.' And children would be satisfied with that, just as we are still satisfied with the fantasies of fields. For contemporary scientists, fields are their Santa Claus.

This is where Yukawa's theory that fields are merely fictions depicting the exchanges of forces by intermediary particles is so comforting.

Possibly we need to re-model our conceptions of electromagnetism on the basis of degrees of freedom and interactions between particles. Even particles do not necessarily exist with any certainty, for the wave-particle duality debates still go on. My own favourite solution to that is the 'theory of the double solution' of

Louis de Broglie, which is often erroneously confused with his early 'pilot wave theory', which he abandoned as long ago as 1927, developing his new theory only in the 1950s and thereafter. His fascinating analysis suggests that particles are better described by the word 'singularities'. However, this is not the place to attempt to summarize the findings of de Broglie, little known though they are outside the French language, considering that so many of his important works have never been translated into English. The one book of his in English which gives the full explanation of de Broglie's position is *Non-Linear Wave Mechanics: A Causal Interpretation*. However, the English language edition omits 'The Theory of the Double Solution' as the subtitle, which appeared beneath the title in the original French edition, thus contributing to the lack of familiarity with the phrase amongst English-speaking readers.[21]

The followers of de Broglie, such as Georges Lochak, are aware of Eddington's views, and in a book written jointly by Borne, Lochak, and Stumpf, have this to say, when complaining that there are some theories that require zero mass for the photon and gauge invariance:

'An old example is a theory of Eddington, based on 16 degrees of freedom and the *exact* formula $1/\alpha = \frac{1}{2}\, 16(16+1) + 1 = 137$ (α = the fine structure constant). Unfortunately, $1/\alpha = 137.036 \ldots$'[22]

Their objection is based upon two points. The first is that: 'electromagnetic gauge invariance, as a law of symmetry, may be proved approximately, not exactly.' The other is that, regarding the mass of the photon,

it 'may be proved experimentally that the mass is *small*, but it cannot be proved that the mass is *exactly zero*, which would be an *algebraic condition*.'[23] To go further into their points would be out of place here. But I must point out that Borne, Lochak, and Stumpf explicitly complained that Eddington's theory must be wrong because the answer to the equation was 137.036 instead of precisely 137. However, as we have seen above, this tiny decimal discrepancy is connected with Martin Rees's universal correction factor in a manner previously explained. This factor was clearly not known to Borne, Lochak, and Stumpf when they were writing in 2001, and it is possible that therefore their objection to the Eddington theory might be something they would wish to reconsider.

The importance of what we are discussing, relating to the number 137, was stressed by Wolfgang Pauli at the end of the address he gave in Stockholm upon being awarded the Nobel Prize for Physics in 1945:

'From the point of view of logic, my report on "Exclusion principle and quantum mechanics" has no conclusion. I believe that it will only be possible to write the conclusion if a theory will be established which will determine the value of the fine structure constant and will thus explain the atomistic structure of electricity, which is such an essential quality of all atomic sources of electric fields actually occurring in nature.'[24]

What is meant by 'degrees of freedom'? This question can be taken back to its root, namely to the most elementary concept of a quantum. I am referring to Planck's constant *h*, which is the *quantum of action*, and which is the same as

saying the most absolute quantum of *angular momentum*. The Planck constant is generally placed over 2π, and dealt with in that form. And that means that a single completed rotation has been introduced. This is rooted in our knowledge that *rotation is absolute*. In this form, the Planck constant is technically known as the 'reduced Planck constant' or the 'Dirac constant'. This enables the Planck constant to be interpreted in terms of radians per second, bringing in angular measure. The implication of this is therefore that this basic constant is really based upon angular momentum, or its angles, conventionally measured in radians. The more directions in which action can take place, which means the more angles (as in degrees of a circle, or radians) at which action becomes possible, the greater the amount of freedom. If we bifurcate the circle and conceive of some entities (let us call them spinning particles), then we end up with spin one half and spin one, don't we? And then we are back with Dirac. And there is no reason why quantum circles should contain 360^o. It might be appropriate if they were divided up by radians instead into either 136^o or 256^o. Then, 'things that happen' in the quantum world such as meson interactions could be measured by angles conforming to integral rather than fractional numerical measures directly relevant to what was going on, i.e., numbers which are actually constants of nature, rather than by a measure which does not 'fit' quantum interactions, such as our arbitrary use of 360^o. And, of course, polar and spherical coordinates must be considered in such cases. We must bear in mind also that if a right angle in Cartesian coordinate computations no longer contains 90^o because we no longer use the 360^o division, then our concepts of sines and cosines and all aspects of angular measure have to be restated. If we were to do this, what unexpected things might 'pop out' at us? That would be an

interesting mathematical and geometrical puzzle for a bored mathematician on some rainy afternoon. The Poincaré spheres which are used as constructed visual aids to trace out the spin vectors of photons on a sphere in closed curves are able to map out the 'polarisation states, or *helicity*, of a photon … (and) for representing the angular momentum of light …'[25] This kind of approach, if extended in appropriate ways, suggests a way forward for exploring particle interactions on the basis of degrees of freedom (for more details of Eddington's views on degrees of freedom see below). Little imagination is needed to intuit the visualisation possibilities which might emerge from pursuing such a project.

We need also to remember in this context that the answer to *rotation that moves forward* as a spinning entity in a linear dimension is our friend the *helix.* And the answer to rotation that stands still is the *torus*. A helix may be viewed as an extended torus. Just as moving vortices can be used for transferring energy (a technology much favoured by the military for decades, and hence largely not public), so helices are used for amplifying energy and charge, as Rudolph Kompfner so brilliantly discovered and his colleague John Robinson Pierce at the Bell Labs even more brilliantly developed and promulgated, in perhaps the greatest advance for the field of electronics in their time.[26] This must be what is really going on at the particle level.

We need to look more closely now at what Eddington said about his 'degrees of freedom theory'. To understand his meaning fully, it is necessary to read his three books which dealt with the subject at such length. But I shall try to give some of the essentials.

Eddington was unhappy with the concept of Fermi-Dirac statistics. He says: 'In what sense are Fermi-Dirac statistics true if they are not obeyed in Nature? … Fermi-Dirac statistics only become important when the particles are crowded together in the degenerate state of matter; and when the Coulomb forces are negligible, the difference between Fermi-Dirac statistics and classical statistics is negligible. Why should we attach so much importance to the way in which Fermi and Dirac expected a crowd of electrons to behave, if their expectation is not fulfilled?'

Eddington is primarily concerned to emphasize the impossibility of ever telling one electron from another, in other words, their indistinguishability. He agrees that Fermi-Dirac statistics are based upon that fact, but they are in his opinion significantly inadequate. He says:

'It is well known that Fermi-Dirac statistics are a consequence of the fact that the particles concerned are indistinguishable from one another. But the origin of the interaction [*between two electrons*] must be the same whether we express it by a new statistics or by the conception of force. If Fermi-Dirac statistics arise from the interchangeability of indistinguishable particles, so also do Coulomb forces. Coulomb energy is therefore energy of interchange. This gives a clear indication of the line to be followed in making a theoretical calculation for its value (1/137r). [*Here, r is the distance two electrons are apart.*] We must accomplish for two charges what Fermi and Dirac appear to have accomplished for a number of charges; that is to say, we must study in detail the

way in which the probabilities connected with a system of two particles are affected by the fact that we cannot distinguish one particle from the other.'

As much as I hate to criticise, even mildly, my old friend David Bohm, I feel called upon to quote a revealing remark which he and Aharonov made in their seminal paper about the Einstein Podolsky Rosen Paradox in 1957. I should explain that they referred to the 'Einstein Rosen Podolsky Paradox', and abbreviated it ERP. Today it is customary to speak of the Einstein Podolsky Rosen Paradox and abbreviate it EPR, having changed the order of the names. (I feel it necessary to point this out for the avoidance of misunderstandings.) Here is the remark from the Appendix to their paper which disappoints me:

'We may take as a typical example a case similar to one already discussed by Furry, namely an experiment in which one particle is scattered by another. *To avoid questions of identity, we suppose that the two particles are different.*'[27] (Italics mine.)

This is unfortunately an example of how 'interchange' can be intentionally ignored 'to avoid questions of identity'. Perhaps such questions should instead be faced.

Obviously, if we are to study the degrees of freedom of a single electron, then we must understand the interactions between *two electrons* in order to have a proper grasp of the matter. As Eddington makes clear, it is no use falling back on Fermi-Dirac statistics in that case, which cannot apply. We have to have the

courage to set those statistics to one side and deal with the real problem head-on, without taking refuge in a statistical blur of large numbers of particles, a procedure which simply fails to address the issue.

Why is Eddington so insistent upon taking this approach? It is because: 'One way of obtaining a definite natural quantity of action is found by considering two elementary particles – two electrons or two protons. They have a mutual electrostatic energy due to the repulsion between them; the quantity of time naturally associated with them is the time that light (or, more appropriately, electromagnetic waves) would take to go from one to the other. Multiplying the energy by this time we obtain the action naturally associated with the pair of particles. It is always the same, whether the two particles are close together or wide apart. … In the study of radiation another natural unit of action appears, namely $h/2\pi$. As we have already mentioned, the modern theory takes this as the unit rather than the unit h originally chosen by Planck. Thus we have two natural units of action, the one arising in electron theory and the other in radiation theory; the second is found to be approximately 137 times the first. … So in all problems involving particles and radiation … we have to do with two systems constructed on a different scale, built out of different-sized atoms of action. The current theory makes no attempt to explore the significance of this difference of scale; it simply accepts it as an empirical fact and introduces an empirical fine-structure constant to allow for it. But this can scarcely be the final limit to progress. We are challenged to find a unified theory of electric particles and radiation in which the electrostatic type of action and the quantum type of action are traced to their source. We shall then be able to understand why one

belongs to an atom 137 times the atom of the other, and indeed to foresee this ratio as clearly as we foresee that the circumference of a circle will be 3.14159 … times its diameter. Towards this development there is one very suggestive clue.'

Eddington then goes into the details of his 136 degrees of freedom, how they are arrived at, and explains that 137 is the 137th degree of freedom, namely that of *interchange*. He says of it: 'The extra degree of freedom in the dynamics of indistinguishable particles explains why the quantum is divided into 137 parts instead of into 136 as we at first expected. The Coulomb energy 1/137r is the energy of the "motion" in the 137th degree of freedom.'

Following a method due to Dirac, Eddington attached *EF* symbols (space tensors of the second rank) to accompany the expression of the Coulomb energy and says: 'How many of these symbols *EF* have we to choose from? The answer is 136. There are 16 varieties of *E* and 16 varieties of *F.* That would give 16^2, or 256, varieties of *EF.* But that assumes that we can tell which of the particles has the *E* and which has the *F.* The distribution is not made that way. It is a feature of quantum theory that the particles are so much alike that we can never tell which is which; and we shall see later that this indistinguishability is actually the source of the energy that we are here studying, so that we must not ignore it here. We have then to make one of 16 possible presents to one particle and one of 16 possible similar presents to the other; but the particles are communists, not believing in private ownership, and it makes no difference which present has gone to which particle. There are 16 ways in which the commune can receive two unlike presents, making 136 in all. The 120 combinations of unlike presents

would be duplicated if we distinguished the recipients, thus raising the number to 256.'

Eddington gives much more explanation, but I cannot repeat all of it here. For a full understanding, it is thus necessary to read Eddington's books.

No serious attempt to follow through fully on Eddington's insights seems ever to have been made.

Eddington's ideas about 'degrees of freedom' are therefore really about the number of possible modes of interaction between two particles. They have 136 different ways in which they can interact with one another, which is a purely mathematical fact. But it has been ignored and, as Eddington says, simply buried away in an empirical fine-structure constant without further examination.

But now, in the light of what we have seen earlier, concerning the importance and manifold recurrences of the number 136, or *ř*, we may have the possibilities of the further insights which Eddington was hoping for as an expansion of his theory. Maybe we should bear in mind the words and concept of 'phase transition' as possibly applying to some of these processes. (Phase transitions were not named or recognised as such in Eddington's day.)

Why is it that apparently no one has pursued this before? I once asked Fred Hoyle, who was Eddington's student at Cambridge, what Eddington was like. He told me he wished he knew. He said that Eddington was a brilliant man

and he longed to be able to ask him countless things, but despite the fact that he was his tutor, he found it difficult to converse with him. Eddington was very formal and personally rather unapproachable. It appears therefore that Eddington was one of those people whose communication abilities came out in his writings, but that he found it difficult to communicate verbally on a person-to-person basis, even with his own student. It is possible that a man who suffered from such personal diffidence was simply incapable of attracting disciples or followers of any kind. In this, he resembles that highly private and diffident French psychologist Pierre Janet, a contemporary of both Freud and Jung whom some believe to have been far more brilliant then they were, but who had not a single disciple, and whose work and whose many profound books therefore fell into a wholly undeserved oblivion, to the great loss of the discipline of psychology.

Eddington's insights also need correlating with what we know of electron pairs, i.e. the two electrons of opposite spin said to inhabit the same energy level according to Pauli's Principle as it is understood today. Are they obeying the 137th degree of freedom, that of interchange, by becoming indistinguishable *except for opposite spin*? Is that what interchange means?

And then there is the phenomenon of Cooper Pairs, discovered in 1956 by Leon Cooper and unknown to Eddington at the time he wrote his book. By defying the Fermi energy, Cooper Pairs seem to be engaging in interchange and thus lending support to the Eddington position. It is said of Cooper Pairs that the interaction can be long-range (a possibility insisted upon by Eddington) and that

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their wave functions become symmetric *under particle interchange* so that they are allowed to be in the same state. Has no one previously noticed that these phenomena accord with Eddington's ideas? Two electrons can thus cease to be fermions and become bosons (or at least 'quasi-bosons' according to Bardeen, though they still together adopt spin 1 whether they are fully paid up bosons or just 'quasis'), when undergoing interchange. Should we not be researching the similarities between the theory of Eddington and the manner in which electrons can *condense by interchange*? Is superconductivity therefore a phenomenon of the *dimension of interchange*? In which case we must look for the number 136 in connection with superconductivity. But as I have already shown above, there is at least one potential occurrence of it in relation to superfluidity, as it expresses a relation between the mean liquification temperatures of Helium^3 and Helium^4 with the boiling temperature of water. (What on earth that really means is another matter, and very hard to imagine, but the number does occur precisely as .0136. So precise is this result that one could even write a very simple equation expressing this relation, which I shall forego here, especially as its significance seems so astonishingly unexpected and inexplicable that it seems better to give further thought to the concepts first, as there may be other terms suggested and hitherto undreamt of relations between apparently separated things involved.) This discussion of superconductivity could lead us next into the fascinating world of phonons and their lattices, rotons, maxons, excitons, biexcitons, and plasmons, into which I do not enter here. But I must point out that these ideas are of the greatest possible importance, and amongst their many profound implications may be a new insight into the central concept of *coherence* in physics, which is

one of the biggest scientific issues of our time. And since so much coherence occurs at low temperatures, we can never know enough about it.

In 1954, Robert Dicke introduced the term 'superradiance' into physics, to describe the emission of light from a gas by molecules which are related to one another by long-range order in a 'coherence field' despite being far apart spatially, and which thus correlate their photon emissions.[28] (Strangely enough, he did not mention the concept of 'subradiance' in that original article, but only in a later publication.) This led to the invention of the maser and the laser. It is well known to all physicists dealing in this area that such coherence phenomena are often associated with Rydberg gases and Rydberg atoms, etc. And the occurrence of 136 with the Rydberg phenomena has already been described above. Clearly, there is promise in looking into this more closely.

Let us consider once again what Nambu and Jona-Lasinio said in their early article:

' … the nucleon mass is a manifestation of some unknown primary interaction between originally massless fermions …'

Is not the changing of a fermion into a boson, which has now been established, something related to this early concept?

Above I mentioned Eddington's 'universal uncertainty constant'. It is now necessary to explain a bit more about it. Although it is customary to speak

of point singularities when speaking of particles and inserting them into equations, Eddington did not go along with this. He insisted that points do not exist (which is true, just as lines of force do not exist, for instance), and that in order to be rigorous, one must take account of what he called 'the uncertainty of the origin', the origin being the origin of two or more axes, which is normally conceived of as a point. Eddington stresses that points are not *observables*. Here is how he begins his book *Fundamental Theory*:

'The quantities occurring in the equations of mathematical physics relate partly to physical objects and partly to a mathematical framework introduced for purposes of reference. Both relativity theory and quantum theory stress the distinction between *observables*, i.e., quantities which could be ascertained by a specified observational procedure, and *unobservables*, i.e. quantities depending partly on the auxiliary mathematical frame which cannot be the subject of actual observation. … We must therefore distinguish between the "physical origin" from which an observable coordinate is measured, and the "geometrical origin" of the auxiliary mathematical frame. … The coordinates postulated in the dynamical equations of wave mechanics must be measured from a physical origin, since they and their conjugate momenta are assumed to be observables, being in fact the typical observables of quantum theory.'[29]

This may all seem rather obvious, but he begins methodically and goes on to draw far-reaching conclusions. He derives a precise value for what he labels by the Greek letter *sigma*, written *σ*, which he claims defines the 'standard

deviation' between the physical origin and the geometrical origin in physics. He naturally gives the mathematics for this and says:

'We shall find that the standard deviation σ … ***puts the scale into*** **the physical frame and everything constructed in the physical frame, whether it be a nucleus, an atom, a crystal or the whole extent of physical space. The main problem in this book is to investigate the way in which the extensions of these various structures are related to σ, and to evaluate the numerical ratios for some of the simpler structures.**

'In stepping over from the geometrical to the physical frame we appear to have freedom of choice of σ. But the freedom is illusory, because σ can only be measured in terms of the physical structures whose scale it has itself determined. To double σ would double all linear constants such as the wave-lengths of the hydrogen spectrum; thus the measure of σ in terms of the wave-length of the H_α line as unit would remain unaltered. … the σ-metric is the recognised metric in relativity theory, quantum theory and practical metrology …'[30]

Sigma **thus defines the uncertainty of the physical reference frame. The concept adopted by Eddington of a 'standard deviation' between the ideal and the real in physics is reminiscent of the Comma of Pythagoras as the 'standard deviation' between the ideal arithmetical series of the octaves and the real arithmetical series of the fifths. I believe both to be different manifestations of the same thing, which is fundamental to the Universe. The ancient musical theorists often referred to this as 'the little gap'.** ***Sigma*** **was Eddington's 'little**

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gap'. As already mentioned above, he computed the precise numerical value of this scalar quantity at 9.604. It is impossible to summarize the extremely lengthy, detailed, and brilliant reasoning and presentation which Eddington gives of his startling insights. However, a few more details must be given of his views on the numbers 136 and 137. Here are some of his comments:

'The ordinary momentum vector and energy tensor have respectively 4 and 10 independent components. But when spin momentum is taken into account, mechanical characteristics are specified by a *complete momentum vector* with 10 independent components or by a *complete energy tensor* with 136 independent components. … Corresponding to the extended vector with 16 components, the extended tensor of the second rank has 256 components. The criterion of chirality (right- or left-handedness) again distinguishes the components which are dormant in the standard achiral environment from the active components. The 10 x 10 combinations of two achiral suffixes and the 6 x 6 combinations of two chiral suffixes give 136 achiral components. These form the complete energy tensor. The 120 combinations in which one suffix is chiral and the other achiral give the dormant components of the extended energy tensor.

'The term "particle" survives in modern physics, but very little of its classical meaning remains, A particle can now best be defined as *the conceptual carrier of a set of variates*. [*Here Eddington adds a footnote*: It is also conceived as *the occupant of a state* defined by the same set of variates.] We shall frequently use the term "carrier" as an alternative to particle.

'The definition includes composite as well as elementary particles. We do not set any limit to the number of variates carried by one particle; but in practice the term will be used only for fairly simple combinations, the more complex carriers being referred to as systems. … The fine-structure constant is conveniently described as the ratio of two units or "atoms" of action. Such natural units are obtained when we multiply a separable element of energy by a time intrinsically associated with it. Corresponding to an atom of particle action there must be an atom of field action 137 times as great…. The factor β – 137/136 will be called the *Bond factor*. [*Eddington's footnote here is*: Attention was first called to it by W. N. Bond, *Nature*, 133, 327, 1934.]' (The Bond he mentions is Wilfrid Noel Bond (1897-1937).)

It is unnecessary to attempt further summary of Eddington's general ideas, which defy summary, and for me to continue would be to do him an injustice. The greatest injustice done to him, however, is that so little attention has been paid to these important insights of his. I do believe them to contain essential truths, and hope that they will receive increased attention in the future. Far from being old and dead ideas, I believe them to hold the promise of explaining much that still puzzles us all about our bewildering Universe.

At this point, before leaving Eddington, however, I must return to musicology to shed further light on ř. In addition to the Pythagorean Comma, which I have discussed earlier, and which yielded the number 136, there is the Pythagorean Limma (Leimma in Greek, meaning 'residue, remnant'). Its value is 256/243. The Pythagorean Limma is the ratio of the seven diatonic semitones

in Pythagorean tuning. The difference between this ratio and the Pythagorean Apotomē (which in Greek literally means 'with the end cut off'), which is the ratio between the five chromatic Pythagorean semitones, gives us the Pythagorean Comma. The fact that the number 256 appears as the numerator in the fraction giving the value of the Pythagorean Limma can hardly be accidental. Probably the best reference work to understand the Limma is the book written by the man who is regarded by many as the supreme musicologist for scales and tones in modern times, Alain Daniélou.[31] Seeing the number 256 appear like this in such close proximity to the number 136, we must consider once again what Eddington told us (above) about these two numbers:

'… the extended tensor of the second rank has 256 components. … The 120 combinations … give the dormant components of the extended energy tensor [*i.e., leaving 136*].'

That is the mathematics of how 256 and 136 relate to particle interactions. The fact that these two numbers occur in ancient Greek Pythagorean, and also ancient Indian, harmonic theory relating to what we today call 'frequencies' (a measurement concept which did not exist in antiquity in our modern sense, but was expressed in other ways, such as by the measurements of string lengths on a monochord, which gave precise numbers), shows that the fundamentals of wavelength, frequency, and vibration interactions have an arithmetical continuity in human knowledge extending back at least 2,500 years. None of this was known to Eddington, who worked purely within the contemporary context.

We now turn our attention to the Boltzmann Constant, yet another constant which deals with the individual particle level. Because it is concerned with relating energy level to temperature for particles, it directly concerns (or even in some way regulates) any possible degrees of freedom either possessed or denied to particles. We can therefore expect this constant to be related to the other constants so far discussed. And in this we are not disappointed. Once again, we do not concern ourselves with the power of ten associated with it, and we concern ourselves solely with the numerical coefficient, which is 138. It is immediately obvious that we have a series of numerical coefficients under discussion: 136, 137, and 138. I shall express this by simple formulae dealing solely with the numerical coefficients, using *alpha* to represent the Fine Structure Constant and for simplicity using k to represent only the Boltzmann Constant's numerical coefficient:

$$\check{r} = 136 \qquad (4)$$

$$\alpha = \check{r} + 1 \qquad (5)$$

$$k = \check{r} + 2 \qquad (6)$$

It is probably better to express (6) in this form below, however:

$$k = (\check{r} + 1) + 1 \qquad (7)$$

The form (7) probably gives a better insight into 'what is going on' with this ascending series.

Here are the formulae shown as a sequence:

$$\check{r} = 136;\ \alpha = \check{r} + 1;\ k = (\check{r} + 1) + 1 \qquad (8)$$

Alternatively, one can express things in a different way:

$$\check{r} = \alpha - 1;\ k = \alpha + 1 \qquad (9)$$

Whichever manner of display one prefers depends upon what one wishes to stress as the central feature of the apparent interrelationships. If you think the Fine Structure Constant should be the centre of attention, then (9) will be what you prefer. It is important to stress these variations of presentation, because we do not yet properly understand the significance of 138. Eddington has gone to great trouble to explain the significance of the relationship between 136 and 137, and exactly what the separate significance of each of those numbers is. But he never considered 138. I do not at present have any extended suggestion as to this. Sceptical persons are free to dismiss the series 136, 137, 138 as fortuitous nonsense, but I do not believe they are wise to do so. We are thus faced with the problem of understanding the series, and that has not yet been done. Throwing out the flimsiest of speculations, I might ask whether 138 represents a reversal of interchange, or otherwise a dual aspect of interchange. In other words, does 138 complete the picture somehow? Wiser persons might know this.

I do note however that, although I said earlier that the ratio of the rest mass of the neutron to the proton was 1.00137, if one continues the decimal, the

latest expression of it appears to be 1.00137841887. Rounded off that would be 1.00138, would it not? Does our number 138 appear therefore in the ratio m_n/m_p? And does that go some way towards answering our question? As the decimal expressions of these ratios seem to evolve over time, as more and more people attempt to refine their exactitude, what do we safely settle for, 137, or 138? In any case, my 'feeling' is that the series 136, 137, 138 has a deep underlying significance, and that its full array of ramifications can be explicated by many persons in the future if they should feel so inclined.

Apart from its appearance in the Boltzmann Constant and its relevance to the Avogadro Number, I have so far found one other particularly strong clue to this problem of the number 138, in the field of electrical studies. Maxwell, Josef Stephan, Max Bernhard Weinstein (who translated Maxwell's great work into German), and Edward Bennett Rosa all concerned themselves with this number, expressed as the decimal fraction 0.1380605. This number was first brought forward by Maxwell as a constant to act as a correction factor for estimating the self-inductance of coils of wire.[32] The correction factor was necessary because the original calculations had been based upon square wires, and to make the calculations realistic and useful, by applying instead to round wires, it was necessary to take circles whose radii were equal to a side of each square describing a section of square wire and calculate the correction for round wires. The side of the square was labelled *D* and the radius of the inscribed circle was labelled *d*. The term *D/d* was then placed into an equation which required the numerical coefficient 0.1380605, as the correction for reducing from a square to a circle. The different scientists worried about whether they should alter this

number somewhat, but the last word on the subject was expressed by Rosa in 1907, who insisted on sticking with 0.1380605.[33] I believe that this gives us the necessary lead to pursue a fuller explanation of the deeper significance of 138, but I have not attempted this as yet.

By coincidence, recently I was looking at a curious little book in German by Wolfgang Ehrenberg with no date, but self-published apparently circa 1980, entitled *Unkonventionelle Physikalische Theorien* (*Unconventional Physical Theories*), out of curiosity. I came across a reference in a paper in that book, which seems to be dated 1961, to 136 in connection with the proton mass. From a brief preface 'Concerning the Author', we are informed that Dr. Wolfgang Ehrenberg was born in Munich in 1909, worked at various German universities and institutes before the War, his wartime activities are not specified, and between 1950 and 1952 he worked for the Argentine Atomic Energy Commission. (Was he one of the Germans who fled to South America after the War, having been involved in Hitler's attempt to build an atom bomb?) He returned to Germany in 1953, and in 1954 he founded an organisation called (in German) the Psychophysical Association. He died in 1989, and there is an archive about him in the Deutsches Museum in Munich which I have not seen. It is clear that he had an instinctive hunch that the mass of the proton was connected somehow with 136 and found a numerical way of producing 1836 from 136. His solution was: $136 \times 3^3/2$.[34] That does indeed yield 1836 (i.e., the product 136 x 9, divided by 2). But that does not constitute a formula, and is merely a way of 'stretching' 136 to 1836 by finding numbers which will do it for him. In other words, 9 and 2 are entirely fortuitous, so that the process has no

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intrinsic value and is an artificial manipulation. I thought it important to make a note of this effort by this German atomic scientist, however, to show that the idea of connecting 1836 to 136 had occurred to someone else, even though he was unable to figure out how to do it without introducing arbitrary numbers to get the answer he wanted. Fortunately my formula (1), above, produces 1836 without the use of any other number than 136 itself, so that there is no arbitrary intrusion of an alien number into the formula, which would have rendered it artificial.

I should now like to consider Planck's constant *h*, the numerical coefficient of which is 6.6260. I have found a formula expressing this coefficient in terms of the familiar numbers 136 and 137, and it is as follows (*h* here standing solely for *the numerical coefficient of* Planck's Constant):

$$(\check{r}/2)^{-1} = h(1.0136 + .0137) \qquad (10)$$

It will be remembered that 1.0136 is the Comma of Pythagoras. I write the formula like this initially so that its composition may be seen clearly. The proper formula is:

$$(\check{r}/2)^{-1} = h(\check{r}^{-2} + (\check{r} + 1)^{-2} + 1) \qquad (11)$$

One may of course transpose this to highlight the value of h in terms of $\check{r}$:

$$h = (\check{r}/2)^{-1} / (\check{r}^{-2} + (\check{r} + 1)^{-2} + 1) \qquad (12)$$

This is preferable. Let us just consider it for a moment. Here we have the numerical coefficient of the quantum of action expressed solely in terms of three separate expressions of ř. Twice we see the addition of unity, a recurring feature in the computations of Eddington, and one of those occurrences gives us Eddington's familiar sequence of 136 + 1 = 137. (See below for a description of the standard "(n + 1) formalism" of quantum theory whereby (+ 1) frequently recurs in quantum formulae.) Can it be a coincidence that Planck's Constant can be expressed in this way? It will be noticed that I have used the expression ř/2, which has the numerical value of 68. What is my reasoning for this? I have not previously used ř/2 in any of the other formulae, nor do I in the ones that follow. I was conscious of the fact that the quantum is the smallest postulated 'singular region' in physics (see discussion of de Broglie below, 'singular region' being a term used frequently by him to refer to particles, and I have borrowed his felicitous expression) and I consider that the quantum of action represents one complete rotation (2π) at the minimum possible 'singular region' dimensionality, which is why physicists customarily use *h*/2*π* in their equations rather than merely h, though perhaps they do not think of it in that way. As 136 is meant to represent the degrees of freedom spherically (or toronally), it seems that there are occasions where it is appropriate to cut it in half. Perhaps we could say that we do this in honour of spin ½, which was discovered by Dirac, and which is probably more fundamental than spin 0 or spin 1, and is more likely to apply to the quantum of action. An electron can have 136 degrees of freedom when interacting (+ 1 for interchange), but possibly when a quantum of action is merely existing and not interacting, we should cut its active degrees of freedom

in half. Perhaps my reasoning is strange and the formula will therefore attract dismay. But the formula is true, whether or not it is at the same time either worthless or useful. And so it would be wrong not to give it, just in case. It gives the correct answer, whatever the rationale behind it, and it effectively derives *h* from *ř*. If we can find something like this which appears to lie behind the Planck Constant, a constant which has since 1900 been assumed to be at the basis of everything, then clearly we have a responsibility to investigate such a matter thoroughly. My efforts represent a mere beginning of such a task.

We come now to the Avogadro Number, or rather I should say, the Avogadro Numbers. For there are three of them, all represented by the same symbol: N_A. They are: 6.022140857 x 10^{23}; 2.73159734 x 10^{26} [*this is a reference to the temperature 273.15 degrees Kelvin, which is equal to zero degrees Centigrade, and I should point out that the numerical coefficient $(2.73)^{-2}$ is the mantissa of the square of the Comma of Pythagoras; 273 is also the ratio of the average pion rest mass to the electron rest mass, as I mentioned above*]; and 1.707248434 x 10^{25}. Which one shall we choose? They are all to different powers of ten and all the numerical coefficients are different. The most popular is 6.022 x 10^{23}, and is the one most commonly used. But how can we deal with such a multiplicity of numbers all claiming to be the same Constant? The only safe thing to do, it seems, is to ignore them all, treat the Avogadro Number as *numerically unspecified*, and deal with it in relationship with other known numbers instead. Fortunately, the Avogadro Number is known to be related to Boltzmann's Constant in the following manner, where *k* is the Boltzmann Constant, *R* is 'the gas constant' and the Avogadro Number is N_A:

$$\boldsymbol{k = R/N_A} \qquad \textbf{(13)}$$

Referring back to (8), by substitution we can say of the Avogadro Number the following:

$$\boldsymbol{N_A \text{ x } ((ř + 1) + 1) = R} \qquad \textbf{(14)}$$

And consequently:

$$\boldsymbol{N_A = R/(ř + 1) + 1} \qquad \textbf{(15)}$$

And since we know the value of the denominator:

$$\boldsymbol{N_A = R/138} \qquad \textbf{(16)}$$

This gives us a new dimensionless numerical value for the Avogadro Number, which is dependent upon *R*, the Gas Constant (technically, 'the Ideal Gas Constant'), *R*. *R* is said to have a value in the ideal gas law in terms of a mole with a coefficient of 8.3144598. However, this refers to work per degree per mole, and there are far more variations of the value of the Ideal Gas Constant than there are of the Avogadro Number, at least 13 in fact. I think that until further progress is made on fixing the Ideal Gas Constant as a reliable dimensionless number, we must look upon the Avogadro Number as being the Ideal Gas Constant (whatever we decide that it is) divided by 138. Further

progress can doubtless be made on this in the future, but at least some clarification seems to have been achieved.

I am dubious about the intrinsic status of gas constants. There are two standard ones, the Ideal Gas Constant and the Special Gas Constant. And as I have just mentioned, there is a multiplicity of numerical values attributed to them. There is no doubt whatsoever that for the purposes of mathematical calculations concerning atomic gases over many decades, they have been invaluable. But what is the real status of atomic gases in the Universe? It is very insignificant. Less that one percent of the gases in the Universe are atomic. The rest are what used to be called *ionised gases*, but since 1928 (when Irving Langmuir introduced the term into physics), they have been called *plasma*. We now know that plasma occurs in gaseous, liquid, solid, and other forms (i.e., dusty, complex, crystal, etc.). And although plasma can contain occasional atoms, it is essentially composed of ions and particles. Bearing this in mind, we have to wonder about what a 'gas constant' really means in the wider scheme of things.

When Max Planck attempted to compute the Avogadro Number, he got a result of 6.175×10^{23}, which he regarded as an improvement on O. E. Meyer's value, calculated in 1899, of 6.40×10^{23}.[35] But the value he had calculated for the Boltzmann Constant differed from today's value. Using that earlier value for *k*, Planck used a formula with a logarithm in it to compute the Avogadro Number. With S being the entropy and W representing the thermodynamic probability (always an integer) of the state, Planck had to use a logarithm in his

computations because the multiplication of probabilities corresponds to an addition of entropies, hence the logarithm. His formula for computing the entropy therefore was:

$$\mathbf{S} = k \log \mathbf{W} \tag{17}$$

This and the many related calculations which Planck was doing at that time led to his concept of *quanta*. But that is another story which has no place here. His formula for the Avogadro Number (which he designated as N) ended up being:

$$\mathbf{R} = k\mathbf{N} \tag{18}$$

He referred to R as 'the absolute gas constant'. In all the related calculations, which involved V for volume and *p* for gas pressure, nothing appeared in the formulae regarding charge or Coulomb forces until he had first found N. Only after finding N did he use that value to try to develop a new method of finding the charge of an electron. Although finding the value of the charge of an electron is a very fine thing to try and do, Planck was thinking in terms of atomic gases, and not ionized gases. Sir William Crookes had already discovered ionised gases in 1879, which he called 'radiant matter' and also 'a fourth state of matter'. We now call what he discovered *plasma*, as I have already said. However I have not seen any evidence that Planck was aware of the work of Crookes, or if he were, that he thought it could have any relevance to what he himself was trying to do. It is best to assume that Planck was entirely

uninformed about what we now call plasma at the time he was doing this work. I do not believe it is worthwhile saying any more about this matter of the status of gas constants here, and I leave it as an open question which could be better answered by professional plasma physicists, whose formulae and equations use such different concepts, and are concerned with such things as Debye Length, for instance. Adjusting the math is a task for a plasma physicist.

One brilliant mathematician and physicist who attempted to rectify what he perceived to be some errors in the way theoretical physics was going was Harry Bateman. In 1934, he published a paper in the *Physical Review* suggesting:

'The electromagnetic equations may be derived from a variational principle by expressing the field in terms of the scalar parameters of a number of constituent fields each of which can be described by means of moving lines of magnetic induction. One set of field equations is then satisfied identically and the other set is derived from the variation principle by varying independently the parameters of all the constituent fields. The advantage of the use of scalar parameters is that the stress tensor occurring the laws of conservation may be obtained by means of a simplified form of Schrödinger's rule. … This means that both electricity and magnetism are present at the same time and move in such a way that a line α = constant β = constant moves with the electricity while a line α' = constant, β' = constant moves with the magnetism. Schrödinger's rule fails to give a tensor depending only on h …'[36]

Expressing fields in terms of scalar parameters is what my discussion is essentially trying to suggest, and an explanation of those scalar parameters in relation to the fundamental constants and their interrelationships is central to this aim. 'Fields' do not really exist, but we talk about 'them' anyway, until such time as we are able to replace them with something which really does exist. We do not yet have the language for this, nor do we have yet the full explication of the concepts which can generate that language. As for the connections between electricity and magnetism, see some fresh suggestions towards the end of this paper.

On this general subject which we have been considering, of degrees of freedom at the particle levels, some basic and simple points should be made, to provide potential guideposts for further thought in the future. First of all there is the Pauli Exclusion Principle. The very fact that it is an exclusion principle should signal to us that it denies degrees of freedom to certain types of particles, which in honour of Enrico Fermi are called fermions. On the other hand, the Pauli Exclusion Principle does not similarly deny degrees of freedom to other types of particles which were named bosons by Paul Dirac, in honour of Satyendra Nath Bose. This difference between fermions and bosons is so obvious to all scientists that it is easy to take it too much for granted, to accept it as 'the way of the world', and not to try to figure it out at a potentially deeper level. After all, has it not been sufficiently dealt with by 'spin'? But what does it really *mean* for it to be dealt with by spin? I suggest that this issue becomes more pressing by the day because of the astounding advances being made at this time in history with the intensive contemporary researches into Bose-Einstein

condensates. So far advanced have these become that they have resulted in the discovery of *a new kind of light*. I cannot go into the details of this extraordinary subject here, but I refer in particular to the work of the Russian scientist Denis Nikolaevich Sob'yanin,[37] who has developed the theory of Bose-Einstein condensation of light. He replied to my query about condensates of light, and whether it is possible to consider them a new state of matter, as follows: 'I do not know whether it is reasonable to name the photon condensate a new state of matter, seeing that it is a kind of light, though new and unusual. But what is important is that Bose-Einstein condensation of light-accumulation of photons at the ground energy level at sufficiently low temperatures is a new physical phenomenon, and the photon condensate is new. It may be better to name the photon condensate a new state of light.'[38] The experimental result on condensates of light has been reported two years earlier in *Nature* by a team from the Institute of Applied Physics at Bonn University.[39]

Considering these and similar developments which are currently taking place, I suggest that a re-think of fermions versus bosons is urgently needed, and that the issues raised here about natural constants and degrees of freedom of 'particles' are more urgent than they have ever been. It seems to me that a certain amount of reconceptualization is called for if we are to overcome some of the roadblocks to our understanding.

Having called attention to some of the issues raised by the Pauli Exclusion Principle in relation to our discussion, I turn now to the strange subject of *effective mass*. This concept is especially important in the world of electronics. As

our discussion is so concerned with the question of mass, clearly if such a bizarre concept as 'effective mass' has arisen, it needs to be faced with courage.

Someone who went to some trouble to try to explain this concept was William Shockley, who shared the 1956 Nobel Prize for Physics with his colleagues John Bardeen and Walter Brattain for the discovery of transistors. The symbol used for 'effective mass' in equations is normally m_{eff} , although Shockley does also use the confusing notation m_n to represent 'the effective mass of an excess electron', despite the fact that m_n is normally used to represent the rest mass of a neutron.[40] Shockley says: '… the application of a force changes the velocity of the electron just as if it had a mass of 1/2*K*. The quantity 1/2*K* for states in the conduction band (or 1/2*K'* for the valence-bond band) is of the same order of magnitude as the mass of the free electron … From the experimental data on silicon and germanium it appears to be quite near the electron mass. For most of the purposes of this chapter not enough use is made of the difference between the effective mass and the mass of the free electron to warrant emphasizing the effective mass at this point in the treatment by giving it a special symbol. We shall, therefore, write 1/2*m* for *K* and proceed accordingly.

'On the basis of this assumption, we see that the response of an electron at the minimum energy point of a Brillouin zone to a force is the same as for a free electron … We shall use this relationship later in connection with conductivity.'[41]

Later Shockley gives 'The well-known formula for the binding energy of a single electron to a nucleus', which I skip here, though it uses his ad hoc

symbol of *m* referred to above, with a superscript star added, and he adds: 'In this formula *e* is the charge on the electron, m^* its effective mass which we take as a free electron mass for purposes of calculation, *h* is Planck's constant, and *Z* is the nuclear charge. For the semiconductor case, *Z* is equal to 1/*k*. … We shall consider first the case of a donor impurity, for example a phosphorus atom inserted in silicon. … an atom like arsenic or phosphorus has one too many plus charges for its share of the four valence bonds. Therefore, it constitutes a positive charge in the lattice. It also brings with it an extra electron which neutralizes this charge but cannot be a part of the valence-bond structure. This extra electron uses wave functions in the conduction band. As we have seen, an electron near the bottom of this band behaves in the crystal in much the same way as an electron behaves in free space, except that the electron in the crystal may have an effective mass different from that of an electron in empty space. In accordance with this picture, we might expect it to behave in the presence of the impurity much as a free electron would behave in the presence of a positive charge. Detailed considerations of the wave mechanical nature of this problem shows that this picture is essentially correct. The electron moving in the conduction band has modes of motion around the impurity atom essentially like those which an electron in free space has around a proton in a hydrogen atom. There are two important differences: In the case of the hydrogen atom, the electron is attracted by the charge of one proton. In the case of the phosphorus impurity, the electron is attracted by the same charge. However, this charge is embedded in a dielectric medium. Because of the polarizability and dielectric constant of the semiconductor, the attraction of the donor impurity for the electron is reduced

… The other essential difference is that the effective mass is not that of a free electron.'[42]

Note the interesting phrase *modes of motion*, which prompts some thought. Modes of motion can lead to or otherwise manifest the loss or gain of dimensions of freedom.

At another point, Shockley is discussing the motion of a free electron along 'the curvature of the free-electron parabola'. I shall abbreviate the quotation which follows:

'The smallness of mass suggests the following problem: Suppose an electron is in a wave-packet state … If a field is applied, the wave-packet will be rapidly accelerated and will move across the crystal in much less time than would a perfectly free electron initially at rest. How can this come about? Where does the electron in the third band get its extra energy? The answer is that it had the extra energy already. State S_1 can be shown to correspond to the superposition of two running waves corresponding to $P = + h/a$ and $P = - h/a$. Thus, although state S_1 has no net electron current, it represents a state of considerable energy. The effect of the applied field is to shift the electron to state S_2 by causing changes in the reflection conditions of the two running waves. As a result, all of the kinetic energy becomes associated with motion towards the right and the electron wave-packet is greatly accelerated. Once state S_2 is reached, however, the transfer of kinetic energy from one motion to another is complete, and the effective mass for additional acceleration has its normal value. …

effective masses of the order of the electron's mass should be the rule rather than the exception in well-bonded semiconductors.'[43]

It does not appear that these mass variations in the electron, resulting in *effective mass*, are thought of as relativistic mass variations, because they are all so short-range. What are we to make of all these arguments? What is really meant by a free electron in empty space? Is there such a thing? Can one justifiably speak of empty space when speaking at the same time of the real world? We must remember that these remarks have been made by someone working with intense engagement with the real world, with real doped crystals in real semi-conductors, with real carefully measured impurities and the most careful measurements and tests. Have particle physicists fully taken on board the semiconductor phenomena encountered on a daily basis by electronics physicists and engineers? Certainly phenomena are going on here which are really variations in degrees of freedom. If a doped impurity in a semi-conductor crystal has this kind of effect on an electron (and on a 'hole', but more of 'holes' in a moment), then what is really happening is a variation in the electron's effectiveness. The concept of *effective mass* could be a conceptual artefact which originates from an attempt to make mathematical sense of observed phenomena. It could just as well be something else. Or the phrase 'effective mass' could be interpreted as an attempt to describe real effects which arise from variations in degrees of freedom of the electron. And as for electrons in empty space, there is no such thing as real empty space, and what is the precise definition of a 'free electron'? Everyone speaks of them all the time, but what are they really?

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Perhaps such an entity can only find proper expression within the terminological and conceptual contexts of degrees of freedom.

Now we turn to 'holes', those curious and bizarre entities which are meant to represent electrons as if they were positively charged. They are admittedly fictitious, just as lines of force are fictitious, and fields are fictitious, and even the concept of 'particle' may even be fictitious. (One wonders what in our thinking is *not* fictitious!) Here is something enlightening that Shockley has to say about them:

'Thus we reach the end of our treatment of an excess electron in the conduction band. The Brillouin zone theory indicates that it behaves in the same way as a free classical electron, provided its energy is always so low that it stays near the bottom of the conduction band. In the next paragraph we shall verify that the behaviour of a hole in the valence-bond band is similarly like that of a positively charged electron.

'As we shall show, the hole is really an abstraction which gives a convenient way of describing the heavier of the electrons. An essential feature in making this abstraction is the fact that a full Brillouin zone with every allowed state occupied can carry no net current. This feature permits the behaviour of the hole to be found directly from the behaviour which the missing electron would have if it were present. … The current due to a hole corresponds to a charge of + *e* moving with the velocity associated with the vacant quantum state …'[44]

Shockley then goes on to speak of *the effective mass for a hole*! He represents it by m_p, which is normally the symbol for the rest mass of a proton, and he presumably intends this in order to stress what we might call the effective positive charge of this electron-which-is-not-an-electron. It was troublesome enough for us worrying about 'effective masses', but now we have to worry about 'effective masses' for holes, when holes admittedly do not even exist. What is the effective mass of something which does not exist? One is reminded of Lewis Carroll's Cheshire Cat, which does not exist and yet which has a grin. All this talk of electrons and holes is the very foundation of electronic theory and electronic technology, and has been found to work when making calculations and analyses of all the wonders of today's technologically advanced world. So what are we to make of it? All the real products of this thinking are there to be seen and are used every day by most of the world's population. Electronics engineers do not have to try very hard to justify their thinking, for the results of such thinking have transformed our world. If therefore some of their thinking is a bit muddy and obscure, and even downright contradictory and illogical from the point of view of pure theory, they have little incentive to change it, because they are surrounded by all its triumphant results and achievements. It is really the job of theoretical physicists to improve the theory and hand it to the electronics people and say: 'Here's a better explanation for what you are successfully doing, just in case you are interested.' And who can blame the technologists if that were to happen and they were to shrug their shoulders and go back to their workshops? But of course, the truth of the matter is that any improvement in

theory will inevitably bring improvements in technology, through a deeper understanding, which will reveal new modes of exploitation and use.

Now we turn to subjects concerning such things as Brillouin zones and Voronoi cells, which are more or less the same thing. Shockley refers to events happening along the edges and surfaces of Brillouin zones, which is where the real action is. These zones and cells are attempts to escape from the limitations of fictitious geometrical points of origin. They are, in effect, ‘singular regions’. The Brillouin zones are conceived of as anchored inside crystal lattices. They and the Voronoi cells are admirable efforts to get closer to the real world. They are other examples of the same struggle to avoid fictitious geometrical points, as we have seen in the work of Eddington, and which was an initial impetus of Roger Penrose’s twistor theory. Louis de Broglie, a friend, colleague and collaborator of Léon Brillouin after whom the Brillouin zones are named, was another who struggled to replace ‘points’ with ‘singular regions’, as we shall now see in more detail.

In pursuing this, I wish now to turn to the wave-particle duality issue, which is surely one of the most debated scientific issues of all time, so that one might wonder why I wish to add to the mountain of comments upon the subject. My intention is to give a brief summary of some of the salient points of the greatly misunderstood ‘theory of the double solution’ of Louis de Broglie, which I believe to be related to the issues raised here so far.

I shall attempt to do justice to some of de Broglie's central points, but it should be understood that no mere summary could ever begin to do justice to the elaborate demonstrations and arguments of de Broglie. In summarizing I shall not give any of his equations or proofs, and anyone wishing for those must consult de Broglie's publications.

De Broglie's position is that real particles exist, and that they are embedded in waves as what he calls 'singular regions'. He gives all the equations to justify the singular regions as legitimate, if alternative, equation solutions. He criticises standard wave mechanics as being based upon linear equations and ignoring the necessary non-linearities. De Broglie refuses to admit the real existence of points. In this, he resembles Eddington and also Roger Penrose, whose twistor theory is based upon this very same denial, as just mentioned. De Broglie points out that the standard wave equations of quantum mechanics are based upon idealised point sources and are linear equations, both of which he believes to be wrong. He insists that the waves in the vicinities of the singular regions become non-linear, and he believes that the edges of the waves also become non-linear, and for that reason they do not spread out indefinitely. He gives all the mathematical evidence for this. He describes in great detail how his positions differ from those of Bohr, Born, Schrödinger, Einstein, Dirac, Pauli, Heisenberg, etc., all of whom he knew personally and with all of whom he had had personal discussions and interactions.

De Broglie's basic approach is to suggest that there are two waves connected with every particle. The real wave he calls the *u* wave. It is this which

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contains the singular region. The fictitious wave which accompanies it (in calculations only) is the *Ψ* wave, which is a more or less standard probabilistic wave familiar from ordinary wave mechanics. He insists that the *Ψ* wave can never be neglected, in order to keep all calculations acceptable to quantum wave mechanics. His theory is thus a bold and innovative theory resting upon standard theoretical conservatism. Temperamentally he was not a 'revolutionary' but an 'evolutionary'. He did not wish to overthrow anything, he wished to push it gently aside and replace it with an improved version. Being an aristocrat who had grown up surrounded by an infinity of *politesse*, he had no wish to be rude. All of his scientific disagreements took place within the drawing room of polite conversation amongst equals. But his ideas were explosive. He suggested, for instance, that all particles of spin higher than ½ were composite particles, and that included photons.

The best way to understand De Broglie's main thesis is to look at his key diagram, reproduced below, which consists of a graph of a typical *u* wave, giving the curve APQB.[45]

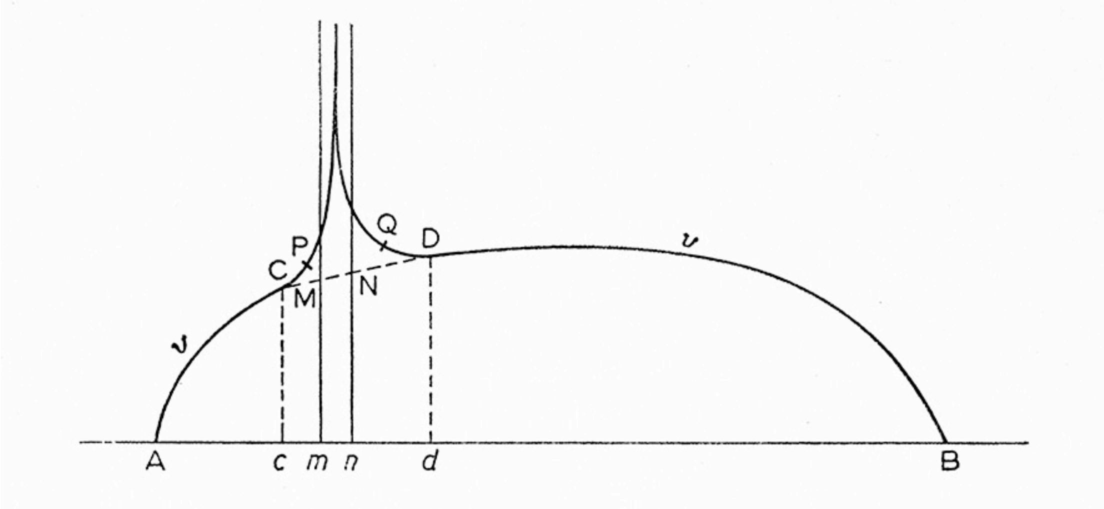

Figure 1. Louis de Broglie's drawing of the curve APQB showing a *u* wave, which rises sharply to show a singular region at the peak seen between *m* and *n*. Reproduced from p. 231 of Louis de Broglie, *Non-Linear Wave Mechanics: A Causal Interpretation*, Elsevier, Amsterdam, 1960.

You will notice a very sharp peak in the graph rising in the middle of the vertical slice marked on the horizontal axis as *mn*. That peak represents the particle. De Broglie divides the *u* wave into three regions. Everything to the left of point C and everything to the right of point D constitutes what he calls 'the external region'. There the equation for the *u* wave is 'practically linear'. Everything between *c* and *m* and between *m* and *d* he calls 'the intermediate region'. There the wave equation is 'approximately linear'. And everything between *m* and *n* is what he calls 'the singular region', with radius r_o ($r_o \leq 10^{-13}$ cm) 'in which the non-linear terms of the equation in *u* are significant'. He envisages the singular region being surrounded by a sphere, and inside that sphere is the singularity. My own preference would be to conceive of it not as a sphere but as a torus with an infinitely small hole, which gives a volume formula

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in which π and 2π both appear, rather than only π, as we find for a sphere. Since de Broglie represents the surface of the sphere by S, and says that 'on S the wave equation is linear, but that *u* increases very rapidly as one penetrates into the sphere', perhaps 'a torus with a vanishingly small hole' would be more appropriate than a torus with an infinitely small hole. A vanishingly small hole would have to be described in equations with derivatives indicating the rate of the vanishing, which might coincide with the rate of the vanishing of the linearity, and perhaps indeed they could be the same thing, or at least represent the same thing. That is just my idea, perhaps worthless. I presume we should have names for these two types of tori. I suggest that a torus with an infinitely small hole could be called a *toron*. A torus with a vanishingly small hole could be called *an asymptotic toron.* I realize that this would mean mixing Latin and Greek terminations, *-us* being Latin and *–on* being Greek. However, since no one teaches the classical languages anymore, presumably no one will know. I won't tell if you don't.

This diagram is an attempt to simplify and display graphically what de Broglie explains in the course of the preceding 230 pages, and thereafter as well. So if the diagram appears simplistic, that is precisely its intention. All the full paraphernalia of equations is arrayed in profusion in his book, together with a vast amount of very clear and cogent commentary and explanation in words, since although he was a brilliant mathematician who loved his equations as much as any physicist, de Broglie came from so highly cultured a background that written and verbal eloquence and clarity of meaning were not only professional but social imperatives for him. He was as profuse with words as Fred Hoyle and

John Wheeler, both of whom were compulsive writers and talkers (the very opposite of poor Dirac, who found it difficult to speak at all, he was so shy and intimidated by the presence of others).

Here is a further attempt by de Broglie to make clear what his three regions are:

'1) a very small spherical region ($r < r_o$) in the immediate vicinity of the origin where the singular term (cos $k'_n r$)/r is entirely preponderant; 2) an intermediate region ($r_o < r < r_1$) where *u_o* and consequently *u* increase rapidly as r diminishes; 3) finally, the region outside a very small sphere of radius r_1 where *u* is effectively [approximately equal to] $C\Psi_n$.'

When explained at very great length by de Broglie, this all makes perfect sense, but to anyone seeing it represented in such a brief manner here for the first time, it may be puzzling. That is entirely my fault, certainly not de Broglie's.

De Broglie speaks of the electron in this manner: '… the singular region that constitutes the electron'.[46] And elsewhere he adds the clarification: 'Each of the singular regions was to be considered as a center of force conditioning the propagation of the wave phenomenon associated with the other particles and, consequently, on the motion of the other particles.'[47]

He thus conceives of the particle formed within the singular region of a *u* wave as a *center of force*. We can either consider *force* as 'something that causes

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a change in the motion of an object' or we can consider a more modern definition as the electron's mass multiplied by its acceleration. But the latter definition falls down when we consider that, as we have seen above, the electron's mass is said to *change relativistically with its acceleration*. If we define the electron's force according to the second definition, we find a double inconstancy, for not only is its velocity changing, but its mass is also changing, You cannot have a reliable derivative according to time passage in such a situation. How, then, can we possibly have a reliable concept of force in these circumstances? Since de Broglie is not speaking here of objects coming into contact, the 'force' to which he refers cannot be what is also known as *contact force*, i.e. the force exerted when two physical objects come into contact with each other, since there is only the one electron on its own. Is the 'force' therefore to be conceived of as 'the interaction between the electron and its environment'? What, then, is its environment? That is also a rather vague notion, to say the least. It is no use saying the force is a vector, since the electron does not need force to have a vector, as it has a vector already simply by moving, thank you very much. De Broglie says the electron's force 'conditions' the motion of other particles by effecting their *waves*. This does not constitute contact between two objects. Heinrich Hertz insisted that:

> **'… force is present only when there are two or more bodies.'[48]**

There have historically been numerous 'force formulae'. Oliver Heaviside in 1889 improved one deriving from Maxwell, Hendrik Lorentz in 1892 derived it as well and added it to the four basic Maxwell-Heaviside equations as a fifth

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basic formula 'to provide a theory in which electromagnetic effects were understood as being due to discrete moving charges.'[49] In the light of de Broglie's ideas, whereby real discrete moving charges do exist within the singular regions of *u* waves, Lorentz might have expected a revival. Lorentz's formula was joined by Alfred-Marie Liénart in 1898 and Emil Wiechert in 1900 to their derivation of Maxwell's theory of the potential due to a moving charge, and this showed 'finally that the resultant force between two moving charges is *not* in general directed along the line joining them.'[50] This leads directly into the vast and complex subject of displacement currents, which we must avoid here, lest the result be a large book. The first Maxwell-Heaviside equation has a displacement term which many people ignore, essentially because they don't know what to think of it, so they just pretend it isn't there. However, they are certainly not wise to do so. The displacement current is fundamental, although some people do not recognise this. Georges Lochak and Harald Stumpf are most insistent about its importance.[51] Georges Lochak was so delighted when he realized that I saw the importance of the displacement current, that he wrote to me in 2012 saying: 'Now I must congratulate you! Until now you are only THE SECOND MAN who understood the interest of all that. The first one is my old friend Harald Stumpf, who is himself a pupil who became a friend of Werner Heisenberg …'[52] Lochak himself is the last surviving pupil of de Broglie, and is President of the Fondation Louis de Broglie in Paris.

The recurrence of (+ 1) in the various formulae presented in this paper may seem unfamiliar to many. However, there is an established tradition in quantum mechanics for such a use of (+ 1) in formulae, according to what is

known as 'the (*n* + 1) formalism'. As Albert Rose expressed it in his 1973 paper 'Classical Aspects of Spontaneous Emission': 'Spontaneous emission is frequently ascribed to the action of zero point fluctuations of an excited electron. … An excited electron relaxes towards lower energy states at a rate, according to the quantum mechanical formalism, proportional to *n* + 1 where *n* is the density of quanta per mode in the field and the term unity is ascribed to "spontaneous emission". … We recognize that, by virtue of the (*n* + 1) formalism, a set of pre-existing zero point oscillations, each having an energy of one quantum, is able *formally* to account for spontaneous emission.'[53] (Rose actually referred to this in the context of arguing for a classical explanation of spontaneous emission.)

According to Fermi-Dirac statistics, the equation for the average number of fermions in an energy state at very low temperature contains the term (+1) in the denominator. According to Bose-Einstein statistics, the equivalent equation for bosons contains the term (-1) in the denominator. (Both equations also contain in the denominator the Boltzmann Constant multiplied by the absolute temperature, in the denominator of an exponent to *e*.)

Léon Brillouin made some profound observations on the (*n* + 1) formalism in a paper he published in 1927, entitled 'A Comparison of the Different Statistical Methods Applied to Quantum Problems'.[54] In that paper he writes it differently, as (1 + *p*), the *p* standing, as is conventional, for momentum. (This differs from the *n* of Rose, which is a term standing for the density of quanta 'per mode within a field'.) The quantities to which unity is added are thus completely different in Rose and Brillouin, and have little to do with one

another at first glance. However, if we probe more deeply into Brillouin's brilliant thinking, we learn some surprising things, and discover that 'everything is not as it seems'. I regard this particular paper by Brillouin as a work of true genius, but not being a French physicist who can pour over the history of French physics literature from 1927 to 2016 with ease, I have no way to be certain of what I suspect, namely that nothing came of Brillouin's insights in that paper, at that time or indeed until now. One caution must be expressed about Brillouin's paper, namely that the terminology used in the translation (and I have not compared it with the original French) is often not the standard terminology of today. He speaks of spatial cells using three different words, as it suits his whim ('cells' included), and one of them, 'compartments', today sounds very odd. He does not use the word 'particles' either, instead preferring 'projectiles'. So in quoting him, I have occasionally substituted a more modern terminology in square brackets to make his meaning clear for us today.

Brillouin's unusual article was written by him as a result of frustration at the 'new statistics' of Fermi-Dirac and of Bose-Einstein. He says he cannot accept that any particles are ultimately indistinguishable from one another, because each one has a different history, and if their histories could be traced back, they could be distinguished. This in itself is perhaps a novel way of looking at things. Motivated by a determination to rethink the whole subject, Brillouin then boldly goes about deriving the necessary particle equations from an entirely different basis of his own. He began by assuming that electrons were indeed identical, but after analysis dismissed it and reverted to a classical theory of particles where 'the [particles[are quite distinct from each other, although they

have similar properties. I shall assume, however, that the [particles] are not independent of each other, so that the probability that an object will land in a [cell] already occupied will depend on the number of previous occupants. With regard to this, I shall make the following very simple assumption: each [cell] has the capacity 1 when empty, and each [particle] has volume *a*; a [cell] which contains one [particle] has still space 1- *a* at its disposal, while one which contains *p* [particles] has free space 1 – *pa*.'

He has defined the states of each of the particles in a six dimensional space x_1, x_2, x_3, p_1, p_2, p_3. Of this phase space, which he calls a 'phase extension', with each particle represented at any given instant by a point, he says: 'The quantum conditions oblige us to assume that this extension is composed of cells [this time he uses the word 'cells'] of finite volume h^3. Two representative points situated in the same cell are supposed to be indistinguishable; they correspond to two [particles] with the same co-ordinates and the same momenta, whose motions will proceed in the same way. The shape of the cells is unknown to us, but this will not inconvenience us in practice. We shall have to evaluate *g*, the number of cells included in a volume Φ of phase extension …(which) is obtained by multiplying *V*, the volume of the configuration extension, by the volume of the momentum extension; in this latter extension the above condition defines a spherical layer of radius *p* and thickness Δ*p*.'

Although Brillouin has said that we do not know the shape of each cell, he proceeds to assume it to be a sphere, and he then specifies that it is within a larger surrounding sphere, the distance between which he calls a 'layer'. In my

copy of this book, an unknown scientist who was its original owner has made interesting marginal notes in pencil, including comments on the equations. At this precise point he has written in the margin: 'Layer?' Since the book still contains the slip in the front saying 'With the author's compliments', this marginal commentator must have been a prominent British scientist of that time. He was clearly struggling with the layer concept.

By modelling two concentric spheres separated by a 'layer', Brilloin has produced a scenario which we may choose to envisage in the following manner: the inner sphere with radius *p* may be viewed as a sphere inscribed within a torus (and situated within its hole). It is easy to define it, because its circumference is given by the inner circle of the torus which encircles the hole in the torus. The outer sphere is circumscribed around the torus, and its circumference is defined by the outer circle of the torus. The 'layer' between the two spheres is thus the space created by the 2π of spinning the torus in a single rotation on an axis running horizontally between two opposite outer edges of the torus, and passing directly through the middle of the hole. If we shrink the 'layer', then the torus becomes an asymptotic toron, as described by me earlier. When $\Delta p = 0$, then we have a toron. By this means, we are able to describe a toron by means of, or in terms of, momenta rather than by means, or in terms of, space coordinates. We should take due note of this advance in defining a fundamental geometrical shape in a new way (dynamically rather than statically), which gives geometry a 'new look'.

Following his strange and original way of thinking, Brillouin ends up by formulating a new theory which then proceeds to yield Planck's formulae. Brillouin's statistical formula differs from the Bose-Einstein formula by including *N*, 'the total number of [particles] … in it …' But he insists: '… circumstances enable us to identify each [particle] … and to distinguish it from its fellows. The complete identity assumed by Bose is a fiction which cannot be realized: each of the objects [i.e., particles] has followed a different route [its 'history'], and this enables us to distinguish it from the others. The photon emitted at a given instant comes from a determinate atom, and the other photons which had accumulated in the same compartment [i.e., cell] had previously quite distinct histories.' So Brillouin emphatically rejected the indistinguishability of bosons, and also for that matter of fermions, even though he accepted the Pauli exclusion principle for them.

Brillouin uses momenta in other unexpected ways, as we see here: 'At a given instant, the atom can exchange a quantum $h\nu$ with only one resonator of radiation, or – which comes to the same – with only one of the h^3 [cells] corresponding to the radiation ν. The probability that the atom should emit a quantum $h\nu$ is proportional to $1 + p$, where p is the number of quanta already in the [cell] h^3.' The probability that the atom should absorb a quantum $h\nu$ is proportional to the number p.'

Having tread his strange path, Brillouin says how curious it is that nevertheless 'there is complete agreement between the point of view which I have developed and the laws of emission and absorption stated by Einstein.' To use a

train analogy, having taken a branch line, he has unexpectedly rejoined the main line. Central to his viewpoint was 'the probability that a [particle] lands in a [cell] which already contains *p* [particles] is proportional to 1 + *p*. … each [cell] has capacity 1 when empty, and each [particle] has still space 1 - *a* at its disposal, while one which contains *p* [particles] has free space 1 – *pa*.'

These ideas of Brillouin give yet a further complexion to the unity which is added to a quantity, and here unity stands for 'a capacity of 1' (or should we say it stands for a hole?) These ideas of Brillouin's should be re-examined fully and extended. He also gives special insights into the nature and relevance of momentum at the particle level. And if we remember what momentum is, namely mass qualified by motion (mass x velocity), then we can see that what Brillouin is really doing is defining various things by mass (albeit the motion of mass), rather than by space. This puts something of a new and provocative light upon the very concept of mass. And as for the (*n* + 1) formalism, here manifest in a different form as a (1 + *p*) formalism, and with the unity which is added seen as a *capacity* rather than as an entity, Brillouin's thoughts could possibly branch out into new realms undreamt of.

Before leaving this subject of adding unity to other quantities, I feel that for the sake of completeness I should record an exceedingly bizarre 'prehistory' for this concept, though it has no connection with physics. In the Middle Ages, the Spanish Jews elaborated a mystical form of Judaism which came to be known as the Kabbalah ('tradition'), often spelled Cabala. From this word our English word *cabal* derives, meaning 'a group of persons engaged in secret

activities'. Something which preoccupied the followers of the Kabbalah was the use of a system called *gematria*. Hebrew and ancient Greek letters both doubled as number systems. Thus, any word in either language automatically also had a numerical value composed of the total number values of its individual letters. *Gematria* is a system whereby special meanings are either decoded or conveyed using the numerical values of words to conceal double meanings which the words *per se* do not have. The Greeks were amused by this and toyed with it, and some of the names of Greek gods when turned into numbers are seen to have geometrical relationships with one another, such as one god being the circumference of a circle of which the diameter is another god, and that sort of thing. An example is that *ho Hēlios* ('the Sun') being equivalent to the diameter of a circle whose circumference is *Poseidōn* (the god of the ocean). Mystical Greeks took this sort of thing seriously, as did Plato. No explicit texts survive describing how these things were done, or who did them (although it is clear that it took place prior to the classical period), so we know little about ancient Greek gematria. A proper study of the differing forms of the divine and heroic names in the different dialects could at least narrow down the origins of this geographically, but no one ever seems to have done this. We do know of various cryptograms which the ancient Greeks used. For instance, Greek playwrights such as Aeschylus, Sophocles, and Euripides tried to defeat rampant plagiarism by secretly writing their names vertically in the first letters of the initial lines of their texts, to act as hidden authorship signatures. The Romans also liked cryptograms, magic squares which spelled names or messages, and so on.[55] And the ancient Egyptians used them as well,[56] had secret coded scripts,[57] and had a passion for concealment in general. But for proper gematria using numerical

values, the only full explanations we possess of the techniques from any earlier culture are those of the mediaeval Jewish Kabbalists and their successors. They believed that they could decode secret messages which had been embedded in the Jewish Torah which were concealed by gematria, and were thus intended only for the wise, who knew how to decode them. They believed that these gave different meanings to numerous parts of the sacred Jewish texts. A fascinating account of various different kinds of gematria is to be found is the lengthy and learned book *The Conciliator* written in 1632 by Menasseh ben Israel, which exists in English translation. Ben Israel was a personal friend of Oliver Cromwell, and persuaded him to allow the Jews to return to England in the 1650s. But that is another story.

And now I shall explain why it is that I am recording these facts here. A fundamental rule of Jewish gematria was apparently discovered empirically. It is called The Rule of *Colel*. The Jews found it necessary sometimes to add one, and sometimes to subtract one, from the numerical values of words decoded by gematria, in order to obtain the true concealed meanings. This happened so often that it was accepted without question by the Kabbalistic rabbis and scholars. They were thus using the (n + 1) formalism to obtain their results (n here standing for any number). This process has been actively practised by the Jews of this persuasion for at least 800 years, but it probably goes back to at least the 7th century BC, for the Kabbalists find that they must do what they call 'adding *colel*' or 'subtracting *colel*' regularly and often in decoding their secret mystical Jewish doctrines from the ancient Torah texts which in their existing form appear to have been codified about that time. (Whether these practices preceded

the textual codifications of the sacred texts by Ezra and Nehemiah after the return from captivity in Babylon is now impossible to determine with certainty, but it is highly likely.) However, that is all I have to say here about this strange ancient practice of using numbers enhanced or diminished by unity.

We have above mentioned the first recognition that the force between two moving particles need not necessarily be directed along the line joining them. We know that spin is a crucial factor in such interactions. Here is an interesting observation by Herbert Corben from his seminal book, *Classical and Quantum Theories of Spinning Particles*:

'The relativistic dynamics of a spinning particle …, even under no external forces, can be surprisingly complex, and a study of some such types of motion, which are formally allowed by the equations of motion, is presented here for the first time. Once we start with a model of a point particle with finite charge, spin, and moments of inertia, we are committed by relativity theory to the strange possibility that its center of mass may lie at another point, and that even if it is moving freely its momentum and velocity are not necessarily proportional to each other. This phenomenon leads to strange motions, as the "particle" may whirl around its own center of mass which in turn moves in response to external forces.'[58]

This sounds very much to me as if Corben's point-like sphere is really a toron, and that the double motion described by Corben is in fact a description of a double-rotation whereby the particle may be conceived of as spinning on its

own axis simultaneously while whirling round toroidally on an additional axis. Corben never considered this toroidal possibility. It naturally never occurred to him, though he noticed the strange details mentioned above. And this has further implications, because if there is a double-motion of all electrons, then the enveloping magnetic fields generated at right angles by electric currents may not at all be at right angles to the second, toroidal, motion, and, frankly, the entire relation between electricity and magnetism may be one which thus conceals a hidden factor. This would mean that we do not really understand electromagnetism properly after all. This is crucial to the science of z-pinches, and could have enormous implications. The toroidal magnetic rings surrounding all electric current flows, which appear to be orthogonal to the current flows and hence a separate phenomenon to the currents, may instead be amplified outer rings of a higher harmonic emanating from existing micro-rings which are flowing within the current, but have never been recognised. In other words, the magnetic orthogonality is not superimposed upon the current at all, and is not a separate thing called magnetism, but emanates from the current in a perfectly normal manner. That would mean that magnetic fields generated by flowing currents are really just second order electric fields. Mystery solved. (This could be a contributing causative factor to the quantisation of flux, the quanta of flux possibly being epiphenomena of a hitherto unrecognised circularity which generates concealed and recurring inner periodicities which thus mandate quantisation.) We would need to reconfigure our thinking and tweak some equations, throwing another 2π in there quickly, just for starters. This would mean having another look at Dirac's equations, at spinors, at all the immense and impressive work that has been done on the concepts and nature of spin in

recent decades, and all sorts of issues. Especially relevant, it seems to me, is the work relating to topology as it applies to electromagnetism, as presented for instance in Terence Barrett's fascinating and inspiring book *Topological Foundations of Electromagnetism*.[59] And let us remember how essential it is, as de Broglie insisted, to use non-linear equations when dealing with the main issues, and to avoid stating key principles based (ostensibly 'for simplicity') upon linear equations. As Steven H, Strogatz has so wisely reminded us: 'Most of everyday life is nonlinear, and the principle of superposition fails spectacularly.'[60]

As for the observations about natural constants, perhaps clever persons could try to combine those insights with the above points, assuming there is any validity at all to this brief excursus.

I add by way of a supplementary note that, although I did not search for macrocosmic occurrences of $\check{r}$ amongst celestial bodies, I could not help but be aware that the sidereal month of our Moon is 27.3 days, the numerical coefficient of which is $(2\check{r} + 1)^{-1}$. If D_M is the lunar day and D_E is the Earth day, then we have the formula:

$$D_M/D_E = (2\check{r} + 1)^{-1} \tag{19}$$

This formula expresses the ratio of the lunar day to the Earth day.

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However, there is a more interesting formula, also based upon ř, which instead of merely giving the numerical information for the value of the ratio, gives some hint of a deeper meaning:

$$D_M/D_E = ((\check{r}^{-4} + 1)^2 - 1) \times 10^4 \qquad (20)$$

This formula also expresses the ratio of the lunar day to the Earth day, but from a differing perspective. It takes account of the fact that the number 273 occurs as the mantissa of the square of the Comma of Pythagoras. Indeed, it is somewhat peculiar, to say the least, that the number 273 can thus be derived from ř in two entirely different ways: on an additive basis and on a multiplicative basis (i.e., from a square). The significance of this depends upon what one thinks of multiplication. Some people maintain that multiplication is merely a fancy form of addition. I am inclined to this view, having studied ancient Egyptian arithmetic to a certain extent. There was a civilisation which lasted for thousands of years, had a spectacular engineering capability, and built all of those magnificent edifices, without possessing a mathematics that included multiplication. The calculations which we get from multiplication they got from extended series of additions. They thus seem to have lived perfectly happily without being able to multiply in their mathematics. Should we therefore view multiplication as being in the same logical category as addition, or should we not? Is it significant that 273 can be derived in the two separate ways from ř, based on the one hand on an addition and on the other hand on a multiplication, or is it not? Is this coincidence important, or is it trivial? I have no answer to this question and content myself with merely posing it for the moment.

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In the case of the lunar and Earth motions, we thus see the familiar coefficients occurring once again in the context of a ratio, and thus giving us a scalar quantity. Is there perhaps some form of self-similarity (to use the fractal phrase) manifesting itself in the Universe whereby the same numerical coefficients continually recur regardless of scale, and can be seen as readily in the relationship between the Earth and the Moon as they can with pi mesons at the particle level? Is the Universe that structured?

Expanded comments and formulae: **Having sent the above text to Dr. Cyril W. Smith, who has worked on electricity, magnetism and water in coherent systems over many decades. I was delighted to receive in reply from him some favourable comments and original calculations. He has been working on something regarding the modulation of scattered light and applied this to planetary distances from the Sun on receiving the paper from me. He was interested in what I had said about the numbers 136 and 137, so he looked for some appearance of one of these numbers in his own calculations, which are entirely different from mine. By his own separate means he found the numerical coefficient 13.7 turning up as a ratio with reference to the planet Jupiter. It is not for me to attempt to explain Cyril's discovery, as it is for him to report it in one of his own publications. But this response encouraged me to look at the planetary mean distances and see what might be there, buried in the data. I was extraordinarily pleased to make some further discoveries, and I believe the one which is most interesting is a formula which I have been able to derive for the**

numerical coefficient of the mean distance of the Earth from the Sun, which is 1.496×10^{11} meters. The formula for this coefficient is as follows:

$$D_E = (\check{r})^{-2} + (\check{r})^{-3} \qquad \textbf{(21)}$$

We thus find that *ř* appears in formulae relating both to the ratio of the Earth's lunar day to that of the Earth's moon, and also to the Earth's mean distance from the Sun. In other words, *ř* appears in formulae both for the Earth's rotation and for the Earth's orbit. But that is not all that became clear.

Assuming the 10^{11} meters as constant for planetary mean distances, one concentrates on the numerical coefficients. The coefficient of the mean distance from the Sun of the planet Venus is 1.082. The ratio of that to the Earth's coefficient is 1.38. The formula is:

$$D_E/D_V = (\check{r} + 2)^{-2} \qquad \textbf{(22)}$$

I then proceeded to make some discoveries relating to Jupiter. I found interesting relationships between Jupiter and Mercury and between Jupiter and Venus. The one between Jupiter and Mercury is very odd indeed, and involved a kind of inverted thinking on my part. I reduced the mean distance of Jupiter from the Sun by one power of ten, obtaining a coefficient that was therefore *smaller* than that of the coefficient of Mercury's mean distance from the Sun. This is significantly counter-intuitive, because Jupiter is so much further away from the Sun than is Mercury, to say the least. But by sliding the decimal one

place in this way, I discovered that if I divided Mercury's coefficient by Jupiter's coefficient reduced by a power of 10 in the manner I just described, the result was 1.36, or $(\check{r})^{-2}$. The formula for this is (using '*Merc*' for Mercury, as I have already used *M* for Moon):

$$D_{Merc} / (D_J)^{-1} = (\check{r})^{-2} \qquad \textbf{(23)}$$

The relationship between Jupiter and Venus is that the ratio for the mean distance coefficients of Jupiter and Venus from the Sun, with the mean distance coefficient of Jupiter once again reduced by a power of ten, is 1.39. The formula for this is:

$$D_V / (D_J)^{-1} = (\check{r} + 3)^{-2} \qquad \textbf{(24)}$$

If we are looking for some sensible pattern here, we see that in expanding outwards from the Sun, the results of the two formulae (23) and (24) expand from $(\check{r})^{-2}$ to $(\check{r} + 3)^{-2}$, which may signify a progression of some kind. And (22) may show another step in this apparent progression by yielding $(\check{r} + 2)^{-2}$. These initial findings seem to suggest that $\check{r}$ may be just as important at the macroscopic scale of our solar system as it is at the particle level. Should this indeed be so, then we face a significant challenge in coming to grips with such an immense level of scaling. It would in that case appear to be impossible to avoid concluding that self-similarity in the fractal sense is active across that entire level of scaling, and that $\check{r}$ is a fundamental building block for the known Universe from the electron to the solar system, and probably at all levels beyond. If so, a

great deal of theoretical thinking is going to have to go into constructing and explaining the operating system hinted at by these findings, and what we have found so far could turn out to be but the beginning of a saga of future discovery.

In closing this paper, I wish to return to some final remarks about the Comma of Pythagoras, which as I mentioned above is 1.0136. Using a tentative and rather cumbersome symbol for it of Com_{pyth}, the formula for it is:

$$Com_{pyth} = (\check{r}^{-4} + 1) \tag{25}$$

I published an explanation of it in my book *The Genius of China*, in connection with my account of the invention of equal temperament in music by the Ming Dynasty Prince Zhu Zai-yü, who published it in his book *New Account of the Science of the Pitch Pipes* in 1584.[61] (It spread to Europe through the Dutch traders, came to the attention of the Dutch scientist and mathematician Simon Stevin, and was eventually adopted by Bach, thereby revolutionising world music.) I wrote that book at the request of my friend Joseph Needham, author of the extensive series of volumes entitled *Science and Civilisation in China*. The part of the book which he praised to me most highly was the section on equal temperament. He confessed that despite great efforts he had never been able to understand equal temperament until he read my explanation of it. Since I regard Joseph as the greatest scholar of the 20th century, this is what I consider to be the highest praise I have ever received for anything I have written. And now, without fully explaining equal temperament here, I must call attention to a few key aspects, which following all the discussion above, may be seen to have

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unexpected relevance beyond the merely numerical. I have already mentioned above that the mathematics of ascending or descending octaves is incommensurable with the mathematics of ascending or descending fifths in music. This is a very shocking and unexpected thing, and people have been being shocked by it now for thousands of years without understanding it. For those who are not musical, I need to say that the *fifth* is the most pleasing harmonic concord to the ear. The Chinese based their music on fifths rather than on octaves. Musical theorists always speak of ascending fifths as 'the spiral of fifths', and they actually draw a spiral to show them graphically ascending (or descending, whichever one prefers). The octaves are generally referred to as ascending in a straight line side by side or within the spiral of the fifths. If you go up (or down) twelve fifths, and up (or down) seven octaves, you end at nearly the same note, but for a very tiny divergence. If you count the frequency arrived at by the spiral of fifths, it is 129.75. But if you count the frequency arrived at by octaves, it is only 128. The frequency by fifths is precisely 1.0136 that of the frequency by octaves. This makes for a horrible discrepancy when it comes to tuning, for instance, a keyboard. The ear can detect it readily. I shall skip all the details of this, which are out of place here. But I wish to call attention to the fact that the fifths are conceived of as spiralling, i.e. progressing in a helix. We thus find helicity involved in this enigma. If you spiral along you seem to get slightly ahead of yourself compared to if you go straight along. The ancients referred to this discrepancy as 'the tiny gap'. Tiny it is, but gap it also is, and its implications are enormous for music. I could give a formula for this, but it is unnecessary, as it is so obvious.

And now I come to a strange discovery I made many years ago, which also manifests the Comma of Pythagoras in a most unexpected way. I can give no scientific explanation for this discovery, which I made empirically. As we all know, the solar year of the Earth is 365.2425 days. Because the year is not a whole number of days, it has caused havoc with calendars for as far back in history as we can trace. I had the idea of seeing what the relationship would be between the true length of the year and an 'ideal' year which would have made everybody so much happier through the ages, of 360 days. To my astonishment, I discovered that the ratio between the two figures is precisely 1.0136. The formula for this, using somewhat cumbersome notation, is:

$$Y_{Esolar} / Y_{Eideal} = (\check{r}^{-4} + 1) \qquad \textbf{(26)}$$

As can be seen from this, I have tentatively named the 360-day year an 'ideal year' not in the technical sense but only as a convenience. Is this a chance identity? How can I possibly justify this? Reverse-engineering my discovery, let us say that I had instead decided to see what an imagined 'ideal year' for the Earth might be if I divided the true year by the Comma of Pythagoras. I would have been most surprised to find myself with the whole number of 360 days. It is tempting to view the spiralling motion of our Earth through space (its elliptical orbit plus the solar system's own 'drift' within the galaxy turns the orbit into an elliptical helix) as analogous to 'the spiral of fifths', and the idealised orbit of a whole number of 360 days as analogous to the octaves. For the two pairs yield the exact same number as a ratio.

I cannot defend this empirical finding, but merely record it.

I hope that these unusual discoveries may be of some use to others.

ROBERT TEMPLE

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[1] Dirac, Paul A. M., 'On the Annihilation of Electrons and Protons', in *Proceedings of the Cambridge Philosophical Society*, Volume 26, 28 October 1929 – 28 July 1930, Cambridge University Press, 1930, p. 361. (This paper was read 19 May 1930.)
[2] Eddington, Sir Arthur, *New Pathways in Science*, Cambridge University Press, 1935. (The text was originally delivered as the Messenger Lectures in 1934.) The key section of the book for this subject is found on pp. 229-54. Since this book is difficult to find today, I should mention that the page numbers are identical in the paperback reprint which was issued as an Ann Arbor Paperback, by the University of Michigan Press, in 1959.
[3] *Ibid.*, p. 242: 'The system of two distinguishable particles is accordingly limited to 136 degrees of freedom.' (He gives a lengthy and detailed explanation for all of this, and explains why 136 is 120 plus 16, and why 16 x 16 resulting in 256 is not valid. 136 can be enlarged to 137 by the addition of the final extra degree of freedom called by him 'interchange'. This explains why the fine structure constant is 137 rather than 136. For some further account of Eddington's opinions in his own words see later in this article.)
[4] Eddington, Sir Arthur, *Relativity Theory of Protons and Electrons*, Cambridge University Press, 1936. Section 12 is devoted to 'The Mass-ratio of the Proton and Electron' and Section 15.8 is entitled 'The Factor $136/137\mu$'.
[5] *Ibid.*, p. 216.
[6] Eddington, Sir Arthur, *Fundamental Theory*, Cambridge University Press, 1953 (the first edition was published in 1948; Eddington died in 1944, leaving the book finished in manuscript except for filling in many of the footnotes, which was done by his friend Edmund T. Whittaker).
[7] Rees, Martin, *Just Six Numbers: The Deep Forces That Shape the Universe*, Weidenfeld & Nicolson, London, 1999, p. 51.
[8] Barbara, André, *The Euclidean Division of the Canon: Greek and Latin Sources*, University of Nebraska Press, 1991, pp. 23 and 28. It should be noted that André Barbara did not carry out the calculation necessary to get the decimal number from the fraction, and that I am the one who has done that. He was therefore unaware of the importance or significance of the fraction given in the text which he edited and

translated. The intention of the text was clearly to conceal the information from the uninitiated, for all that the text says is that the number 531,441 is greater than twice 262,144. Unless you already knew (as I did, fortunately) what the author was talking about, it would be quite impossible to know what he meant, or have the slightest grasp of the importance of his statement. The editor and translator of the text was himself unaware of what was really being said. Working with these sources is a difficult business, and few are those who could realize from the following translated text what the number given in the text was expressing: 'The diapason is less than six tones. For the diapason was proved to be duple, and the tone sesquioctave. Six sesquioctave intervals are greater than a duple interval, therefore the diapason is less than six tones. … The tone will not be divided into two equal tones nor into more. … Therefore the tone will not be divided into equal intervals.' (A diapason is an octave, and the statement that a tone will not be divided into two equal tones refers to the fact that there is no true semi-tone in nature; semi-tones are an artificial invention originated in China by Zhu Cai-Yü in 1584, for which see pp. 234-9 of my book *The Genius of China*, 1986, reprinted numerous times and the revised and latest edition of which is 2006.)

[9] This is discussed *passim* and at great length in his *Fundamental Theory*, *op. cit.*.

[10] Nambu, Yochiro and Jona-Lasinio, Giovanni, 'Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity', in the *Physical Review*, Volume 122, Parts 1 and 2, 1961, p. 346, column b.

[11] Heisenberg, Werner, *Natural Law and the Structure of Matter* (an address delivered in Athens on 3 June 1964), The Rebel Press, London, 1970, p. 31.

[12] Jammer, Max, *Concepts of Mass in Contemporary Physics and Philosophy*, second revised edition, Princeton University Press, 2000.p, p.166. This quotation is from the penultimate paragraph of the book.

[13] Hesse, Mary, *Forces and Fields: The Concept of Action at a Distance in the History of Physics*, Thomas Nelson and Sons Limited, London and Edinburgh, 1961, pp. 277-8 and n. 2.

[14] Mitra, Asoke Nath, and Saxena, R. P., 'Meson-Meson Interaction in the Bethe-Salpeter Approximation', in *Physical Review*, Volume 108, Number 4, November 15, 1957, p. 1083.

[15] Fermi, Enrico, *Elementary Particles*, Oxford University Press, 1951, p. 53.

[16] *Ibid*., pp. 50-1.

[17] Bennett, Willard Harrison, 'Magnetically Self-Focusing Streams', in *Physical Review*, Volume 45, 1934, pp. 890-7.

[18] Velikovich, Clark, Davis, Chong, Deeney, Coverdale, Ruiz, Cooper, Franklin, and Rudakov, 'Z-Pinch Plasma Neutron Sources', US Naval Research Laboratory, Washington DC, 'approved for public release', March 24, 2006.

[19] Heil, Herman Gustavus, and Bennett, Willard Harrison, *Fundamental Principles of Physics*, Prentice-Hall Inc., New York, 1939, p. 433. (Bennett is the same Bennett after whom 'the Bennett Pinch', formerly the 'Z-Pinch', is named.)

[20] Bridgman, Percy Williams, *The Logic of Modern Physics*, the Macmillan Company, New York, 1927, pp. 57-8, 132-137.

[21] De Broglie, Louis, *Non-Linear Wave Mechanics: A Causal Interpretation*, Elsevier Publishing Company, Amsterdam, 1960. This English translation omits from the title page the crucial subtitle 'The Theory of the Double Solution' which appeared beneath the title of the original edition: *Une Tentative d'Interprétation Causale et Non*

Linéaire de la Mécanique Ondulatoire (La Théorie de la Double Solution), Gauthier-Villars, Paris, 1956.
[22] Borne, Thomas, Lochak, Georges, and Stumpf, Harald, *Nonperturbative Quantum Field Theory and the Structure of Matter*, Kluwer Academic Publishers, Dordrecht, 2001, p. 23, n.4.
[23] *Ibid.*, p. 23.
[24] Pauli, Wolfgang, *Exclusion Principle and Quantum Mechanics*, Lecture Given in Stockholm after the Award of the Nobel Prize of Physics 1945, Editions du Griffon, Neuchatel, Switzerland, 1947, pp. 50-1.
[25] Barrett, Terence William, *Topological Foundations of Electromagnetism*, World Scientific Press, Singapore, 2008 (reprinted 2009), p. 33.
[26] Kompfner, Rudolf, *The Invention of the Traveling-Wave Tube*, San Francisco Press, San Francisco, 1964. Pierce, John Robinson, *Traveling-Wave Tubes*, D. van Nostrand Company, Princeton, 1950.
[27] Bohm, David, and Aharonov, Yakir, 'Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky', in *Physical Review*, Volume 108, Number 4, November 15, 1957, p. 1075.
[28] Dicke, Robert Henry, 'Coherence in Spontaneous Radiation Processes', in *The Physical Review*, Volume 93, Second Series, Number 1, January 1, 1954, pp. 99-116.
[29] Eddington, *Fundamental Theory, op. cit.*, pp. 1-2.
[30] *Ibid.*, pp. 4-7.
[31] Daniélou, Alain, *Introduction to the Study of Musical Scales,* The India Society, London, 1943, *passim.*
[32] Maxwell, James Clerk, *A Treatise on Electricity and Magnetism*, Volume II, Section 693, Clarendon Press, Oxford, 1873. I do not have this edition, and only have the third edition of 1892, in which this number was removed.
[33] Edward Bennett Rosa, 'On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance', in *Bulletin of the Bureau of Standards*, US Department of Commerce and Labor, Washington, Volume 3, Nos. 1-4, 1907, pp. 1-41.
[34] Ehrenberg, Wolfgang, *Unkonventionelle Physikalische Theorien* (*Unconventional Physical Theories*), self-published by the Psychophysical Association of Munich, no date but apparently circa 1980, p. 32.
[35] Whittaker, Sir Edmund, *A History of the Theories of Aether & Electricity*, Volume Two, *The Modern Theories*, 1953, reprinted as a Harper Torchbook, Harper & Brothers, New York, 1960, p. 85.
[36] Bateman, Harry, 'Sidelights on Electromagnetic Theory', in *Physical Review*, Volume 45, Second Series, January-June, 1934, pp. 721-3.
[37] Sob'yanin, Denis Nikolaevich, 'Theory of Bose-Einstein Condensation of Light in a Microcavity', in *Bulletin of the Lebedev Physics Institute*, Vol. 40, Number 4, Allerton Press Inc., 2013, pp. 91-96. (The original Russian publication of this article was in *Kratkie Soobshcheniya Po Fizike*, Vol. 40, Number 4, 2013, pp. 15-24.)
[38] Personal communication, 16 June 2015.
[39] Klaers, Jan, Schmitt, Julian, Vewinger, Frank, and Weitz, Martin, 'Bose-Einstein Condensation of Photons in an Optical Microcavity', *Nature*, Volume 468, 25 November 2010, pp. 545-8.
[40] Shockley, William, *Electrons and Holes in Semiconductors*, D. Van Nostrand Company, Toronto, New York, and London, 1950, second printing (text unchanged), 1951, p. 175.

[41] *Ibid.*
[42] *Ibid.*, pp. 224 and 223-4.
[43] *Ibid.*, pp. 397-8.
[44] *Ibid.*, pp. 177-8.
[45] De Broglie, *op. cit.*, Figure 11 on page 231.
[46] *Ibid.*, p. 283.
[47] *Ibid.*, p. 141.
[48] Quoted in Hesse, *op. cit.*, p. 212.
[49] *Ibid.*, p. 219.
[50] *Ibid.*
[51] See Lochak, Georges, Stumpf, Harald, and Hawkes, Peter W., *Advances in Imaging and Electron Physics: The Leptonic Magnetic Monopole: Theory and Experiments*, Elsevier, Amsterdam, 2015, *passim.*
[52] Personal communication, 14 May 2012.
[53] Rose, Albert, 'Classical Aspects of Spontaneous Emission', in Hermann Haken and Max Wagner, eds., *Cooperative Phenomena*, Springer-Verlag, Berlin, 1973, pp. 303-7.
[54] Brillouin, Léon, 'A Comparison of the Different Statistical Methods Applied to Quantum Problems', English translation appearing in Louis de Broglie and Léon Brillouin, *Selected Papers on Wave Mechanics*, Blackie & Son, London and Glasgow, 1928, pp. 139-151. The original French paper appeared in the *Annales de Physique*, Volume 7 (1927), p. 315.
[55] One particularly interesting book dealing with these matters is Moeller, Walter O., *The Mithraic Origin and Meanings of the Rotas-Sator Square*, Brill, Leiden, 1973.
[56] One useful reference is Zandee, Jan, *An Ancient Egyptian Crossword Puzzle. An Inscription of Neb-wenenef from Thebes*, Leiden, 1966.
[57]The priests sometimes had hieroglyphic inscriptions carved on the walls of the more secret parts of their temples in what Egyptologists call 'enigmatic script'. They changed the meanings of the hieroglyphic signs in a systematic manner so that temple servants and scribes could not read them without knowing the 'code', and this main form of enigmatic script has been deciphered by modern scholars, one of the most prominent and successful of whom was the Dutch Egyptologist Adriaan de Buck (1892-1959). However, there are a few surviving texts which were written in another form or forms of rarer enigmatic script, and these have not yet been decoded. Nor has a study ever been done to my knowledge as to whether double meanings might be conveyed in some religious texts by the use of two layers of overlapping script. However, gematria was impossible for the ancient Egyptians, because they had an entirely separate numerical system which did not involve the use of syllabic hieroglyphs, but had its own unambiguous number signs.
[58] Corben, Herbert Charles, *Classical and Quantum Theories of Spinning Particles*, Holden-Day, San Francisco, 1968, pp. vii-viii.
[59] Barrett, *op. cit.*
[60] Strogatz, Steven Henry, *Nonlinear Dynamics and Chaos*, Westview Press, Perseus Books, Cambridge, Massachusetts, 1994, p. 9.
[61] Temple, Robert, *The Genius of China: 3000 Years of Science, Discovery & Invention*, expanded edition, André Deutsch, London, 2006, pp. 234-9. The book was originally published in 1986 under the title *China: Land of Discovery and Invention*, but by 1987 all English language editions had been retitled *The Genius of China.* The

book has been published in 43 languages, many of those by UNESCO, and has appeared in four different Chinese translations.